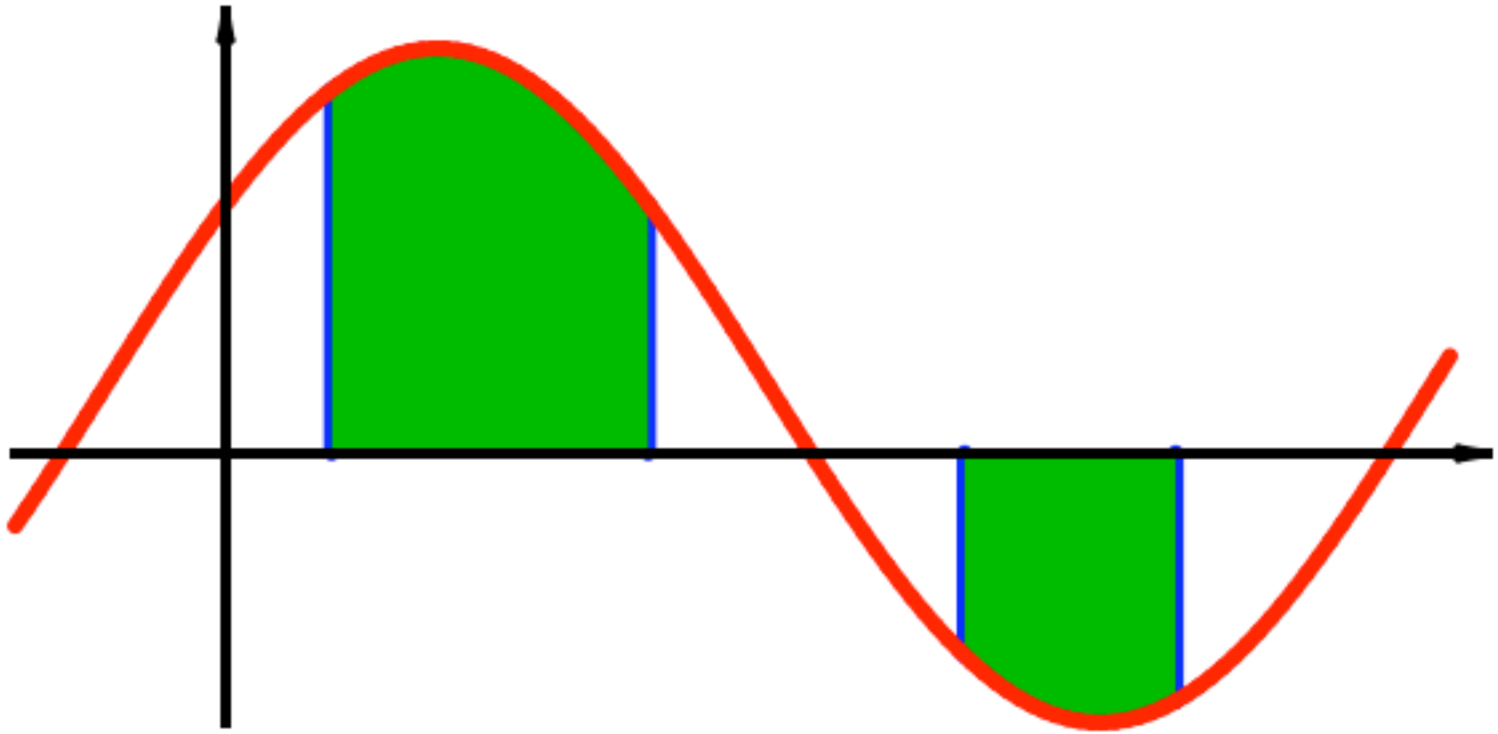

Modul 108 Integration

Integration

Umkehrung des Ableitens

integral: vollständig

summiert

“über alles”

Hauptsatz

Integrationstechniken (nächster Modul)

Integrales Lebenseinkommen

Integrale Sicht:

Während unseres Lebens

gucken wir 12 Jahre in die Glotze

Integrale Sicht:

Während unseres Lebens

gucken wir 12 Jahre in die Glotze

haben wir 7 Wochen Mathe

Integrale Sicht:

Während unseres Lebens

gucken wir 12 Jahre in die Glotze

haben wir 7 Wochen Mathe

küssen wir 2 Wochen lang

$F(x)$ Stammfunktion von $f(x)$:

$$F'(x) = f(x)$$

$$f(x) = 3x^2$$

$$\text{Stammfunktion}: \ F(x) = x^3$$

$$f(x) = 3x^2$$

Stammfunktion: $F(x) = x^3$

Weitere Stammfunktionen:

$$F(x) = x^3 + 12$$

$$F(x) = x^3 - \pi$$

$$f(x) = 3x^2$$

Stammfunktion: $F(x) = x^3$

Weitere Stammfunktionen:

$$F(x) = x^3 + 12$$

$$F(x) = x^3 - \pi$$

Allgemein:

$$F(x) = x^3 + C$$

Integrationskonstante

$$F(x) = x^3 + 12$$

$$F(x) = x^3 - \pi$$

Allgemein:

$$F(x) = x^3 + C$$

Integrationskonstante

$$F(x) = x^3 + 12$$

$$F(x) = x^3 - \pi$$

Von Fall zu Fall verschiedene Integrationskonstanten

Allgemein:

$$F(x) = x^3 + C$$

Integrationskonstante

$$F(x) = x^3 + 12$$

$$F(x) = x^3 - \pi$$

Von Fall zu Fall verschiedene Integrationskonstanten

Allgemein:

$$F(x) = x^3 + C$$

Integrationskonstante

„Konstant“ meint hier: Hängt nicht von *x* ab

Unbestimmtes Integral von f
= Menge aller Stammfunktionen von f.

Unbestimmtes Integral von f
= Menge aller Stammfunktionen von f.

Schreibweise:

$$\int f(x)\,\mathrm{d}x$$

Unbestimmtes Integral von f
= Menge aller Stammfunktionen von f.

Schreibweise:

Integrand

$$\int f(x)\,\mathrm{d}x$$

Unbestimmtes Integral von f
= Menge aller Stammfunktionen von f.

Schreibweise:

Integrand

$$\int f(x)\,\mathrm{d}x$$

Integrationssymbol

Unbestimmtes Integral von f
= Menge aller Stammfunktionen von f.

Schreibweise:

Integrand

$$\int f(x)\,\mathrm{d}x$$

Integrationssymbol

x = Integrationsvariable

Beispiele:

$$\int 3x^2 \,\mathrm{d}x = x^3 + C$$

$$\int 3t^2 \,\mathrm{d}t = t^3 + C$$

$$\int 3☺^2 \,\mathrm{d}☺ = ☺^3 + C$$

CAS

```
> Int(3*x^2, x)=int(3*x^2, x)+C;
```

$$\int 3\,x^2\,dx = x^3 + C$$

CAS

```
> Int(3*t^2,t)=int(3*t^2,t)+C;
```

$$\int 3\,t^2\,dt = t^3 + C$$

CAS

```
> Int(3*x^2,t)=int(3*x^2,t)+C;
```

$$\int 3\,x^2\,dt = 3\,x^2\,t + C$$

Beispiele: $\int x^s \, \mathrm{d}x = \frac{1}{s+1} x^{s+1} + C \quad ; \quad s \neq -1$

Beispiele: $\int x^s \, \mathrm{d}x = \frac{1}{s+1} x^{s+1} + C \quad ; \quad s \neq -1$

$$\int x^5 \, \mathrm{d}x = \frac{1}{6} x^6 + C$$

Beispiele: $\int x^s \,\mathrm{d}x = \frac{1}{s+1} x^{s+1} + C \quad ; \quad s \neq -1$

$$\int x^5 \,\mathrm{d}x = \frac{1}{6} x^6 + C$$

$$\int \frac{1}{x^5} \,\mathrm{d}x = \int x^{-5} \,\mathrm{d}x = \frac{1}{-4} x^{-4} + C = -\frac{1}{4} \frac{1}{x^4} + C$$

Beispiele: $$\int x^s \, \mathrm{d}x = \frac{1}{s+1} x^{s+1} + C \quad ; \quad s \neq -1$$

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$$\int \frac{1}{x^5} \, \mathrm{d}x = \int x^{-5} \, \mathrm{d}x = \frac{1}{-4} x^{-4} + C = -\frac{1}{4} \frac{1}{x^4} + C$$

$$\int \frac{1}{\sqrt[3]{x^2}} \, \mathrm{d}x = \int x^{-\frac{2}{3}} \, \mathrm{d}x = \frac{1}{\frac{1}{3}} x^{\frac{1}{3}} + C = 3\sqrt[3]{x} + C$$

Beispiele:

$$\int x^s \,\mathrm{d}x = \frac{1}{s+1} x^{s+1} + C \quad ; \quad s \neq -1$$

$$\int x^5 \,\mathrm{d}x = \frac{1}{6} x^6 + C$$

$$\int \frac{1}{x^5} \,\mathrm{d}x = \int x^{-5} \,\mathrm{d}x = \frac{1}{-4} x^{-4} + C = -\frac{1}{4} \frac{1}{x^4} + C$$

$$\int \frac{1}{\sqrt[3]{x^2}} \,\mathrm{d}x = \int x^{-\frac{2}{3}} \,\mathrm{d}x = \frac{1}{\frac{1}{3}} x^{\frac{1}{3}} + C = 3\sqrt[3]{x} + C$$

Sonderfall:

$$\int \frac{1}{x} \,\mathrm{d}x = \ln\left(|x|\right) + C$$

Beispiele:

$$\int \sin(x)\,\mathrm{d}x = -\cos(x) + C$$

$$\int \cos(x)\,\mathrm{d}x = \sin(x) + C$$

$$\int \mathrm{e}^{x}\,\mathrm{d}x = \mathrm{e}^{x} + C$$

Rechenregeln

$$\int \left(f(x) + g(x) \right) \mathrm{d}x = \int f(x)\, \mathrm{d}x + \int g(x)\, \mathrm{d}x$$

Wo bleibt das C ?

$$\int \lambda f(x)\, \mathrm{d}x = \lambda \int f(x)\, \mathrm{d}x$$

Beispiel: $\int 3x^2 - 8x + 7 \,\mathrm{d}x$

Beispiel: $$\int 3x^2 - 8x + 7 \, \mathrm{d}x = \int 3x^2 \, \mathrm{d}x + \int -8x \, \mathrm{d}x + \int 7 \, \mathrm{d}x$$

Beispiel: $$\int 3x^2 - 8x + 7 \,\mathrm{d}x = \int 3x^2 \,\mathrm{d}x + \int -8x \,\mathrm{d}x + \int 7 \,\mathrm{d}x$$

$$= 3\int x^2 \,\mathrm{d}x - 8\int x \,\mathrm{d}x + 7\int \,\mathrm{d}x$$

Was ist hier der Integrand?

Beispiel: $\int 3x^2 - 8x + 7 \, \mathrm{d}x = \int 3x^2 \, \mathrm{d}x + \int -8x \, \mathrm{d}x + \int 7 \, \mathrm{d}x$

$$= 3\int x^2 \, \mathrm{d}x - 8\int x \, \mathrm{d}x + 7\int \, \mathrm{d}x$$

$$= 3\frac{x^3}{3} - 8\frac{x^2}{2} + 7x + C$$

Beispiel: $$\int 3x^2 - 8x + 7\,\mathrm{d}x = \int 3x^2\,\mathrm{d}x + \int -8x\,\mathrm{d}x + \int 7\,\mathrm{d}x$$

$$= 3\int x^2\,\mathrm{d}x - 8\int x\,\mathrm{d}x + 7\int\,\mathrm{d}x$$

$$= 3\frac{x^3}{3} - 8\frac{x^2}{2} + 7x + C$$

$$= x^3 - 4x^2 + 7x + C$$

Bestimmtes Integral

integral	$\hat{=}$	ganz
		vollständig
		aufs Ganze gesehen
integer	$\hat{=}$	ganz
		ethisch korrekt

Wie viel Wasser fließt den Rhein hinunter?

Wie viel Wasser fließt den Rhein hinunter?

jetzt? (Momentanproblem)

$f(t)$ Durchflussmenge pro Zeiteinheit, z. B. pro Sekunde

im Jahr? (Integralproblem)

$$\sum_{\substack{\text{alle Sekunden} \\ \text{des Jahres}}} f(t_i)\Delta t = \int_{1.\,\text{Jan}}^{31.\,\text{Dez}} f(t)\,\mathrm{d}t$$

Wie weit kommen wir?

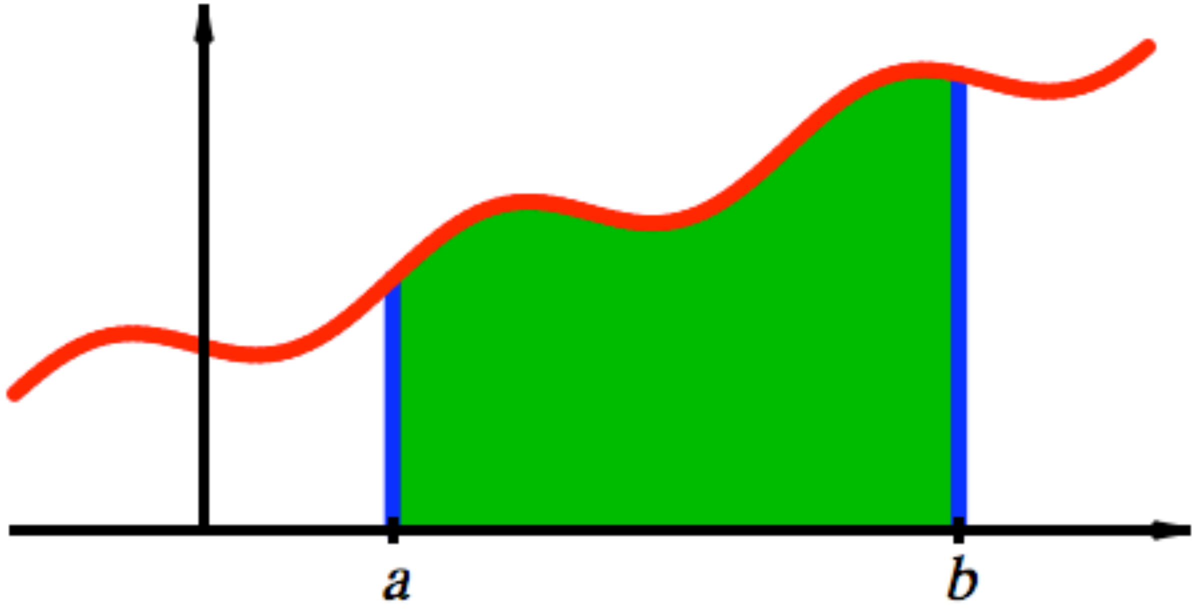
jetzt? Meter pro Sekunde
Momentangschwindigkeit
$v(t)$

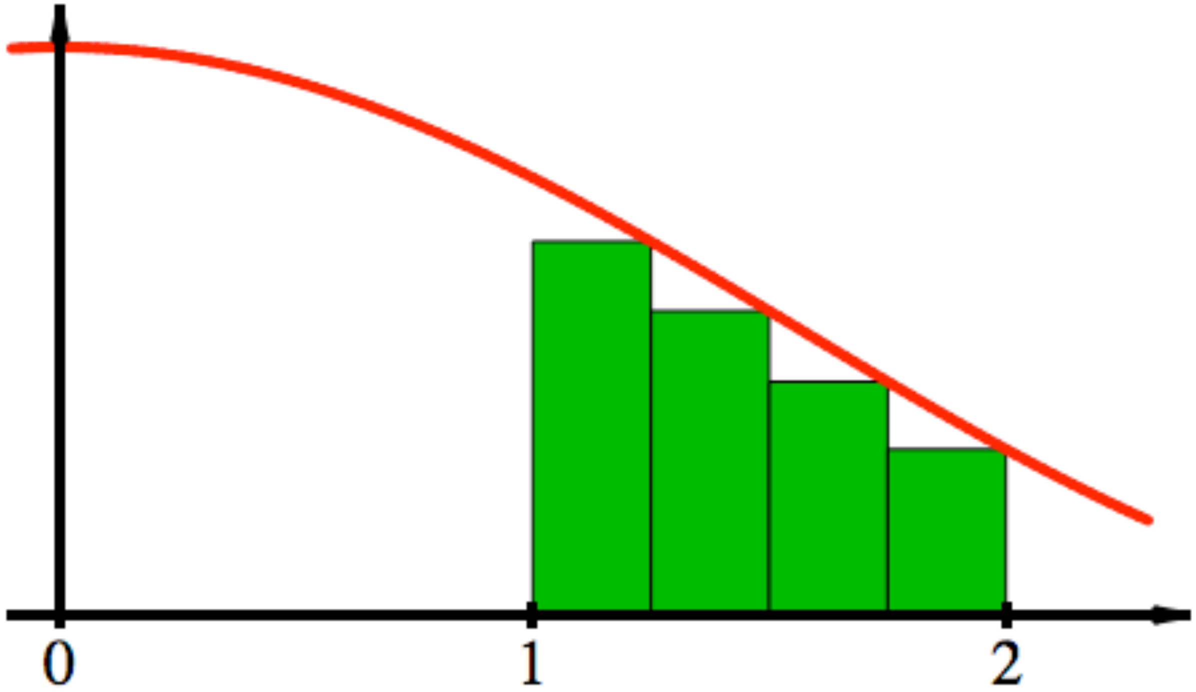
insgesamt? integrale Wegstrecke

$$s = \int\limits_{\substack{a \\ \uparrow \\ \text{Startzeit}}}^{\substack{\text{Zielzeit} \\ \downarrow \\ b}} v(t)\,\mathrm{d}t$$

„Fläche unter der Kurve“

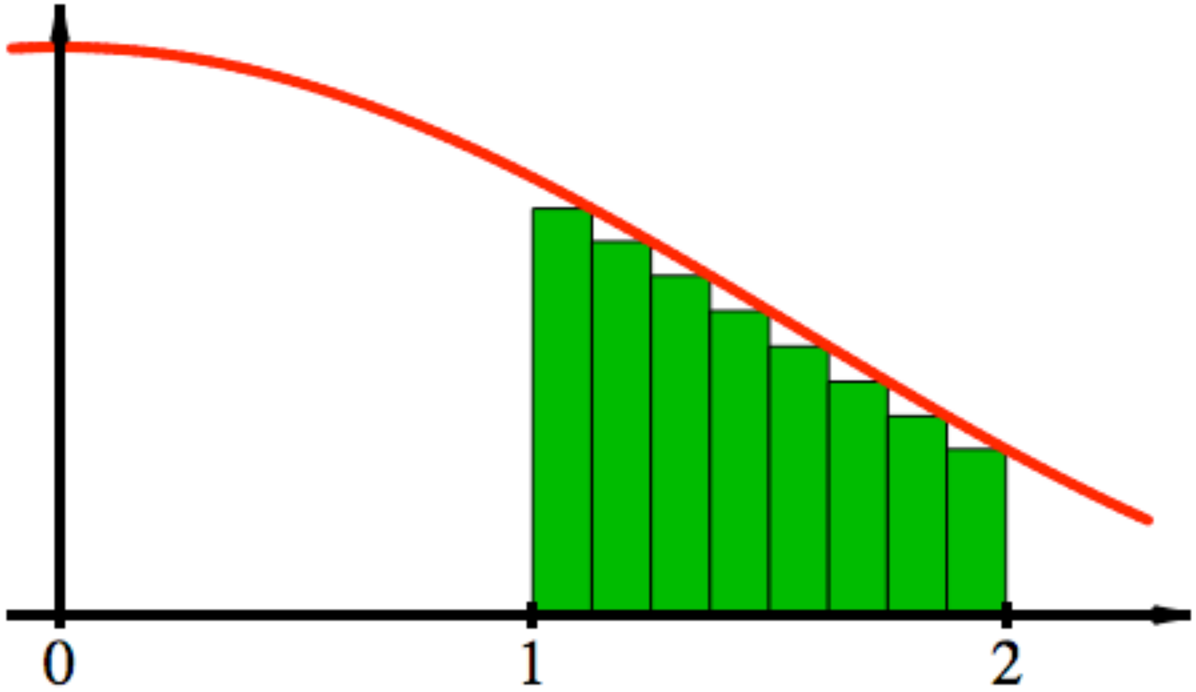
Ein intelligenter Blödsinn



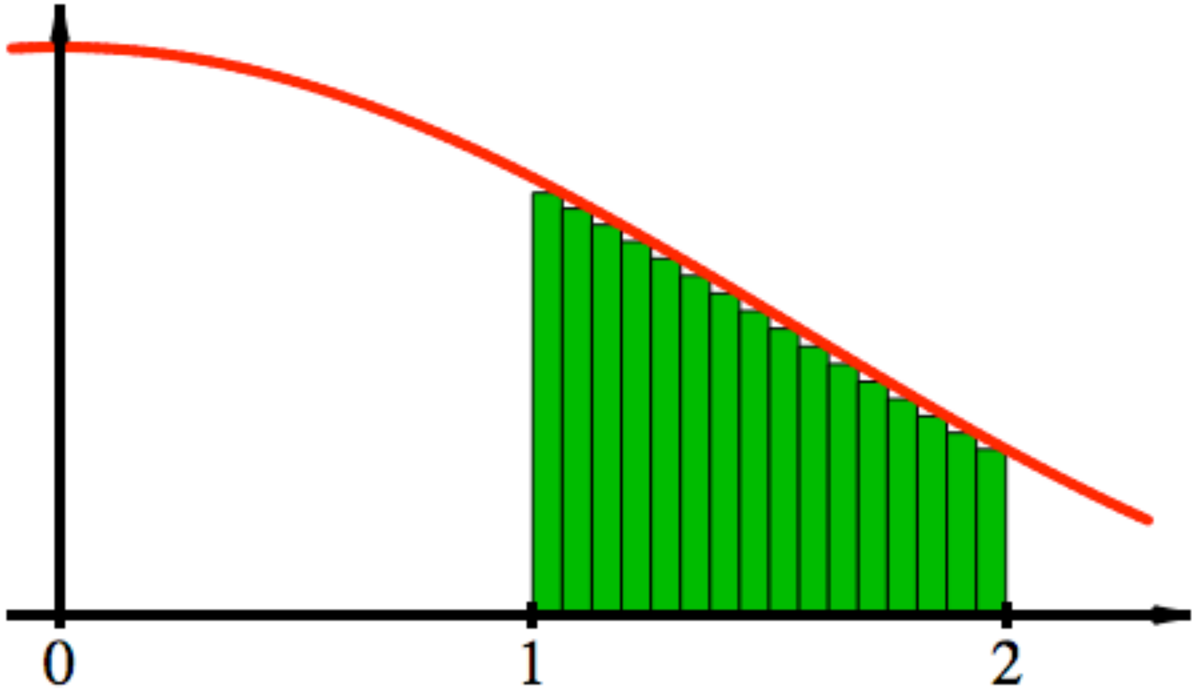


$$A \approx f\left(1+\frac{1}{4}\right)\cdot\frac{1}{4} + f\left(1+\frac{2}{4}\right)\cdot\frac{1}{4} + f\left(1+\frac{3}{4}\right)\cdot\frac{1}{4} + f\left(1+\frac{4}{4}\right)\cdot\frac{1}{4}$$

$$= \sum_{k=1}^{4} f\left(1+\frac{k}{4}\right)\cdot\frac{1}{4}$$

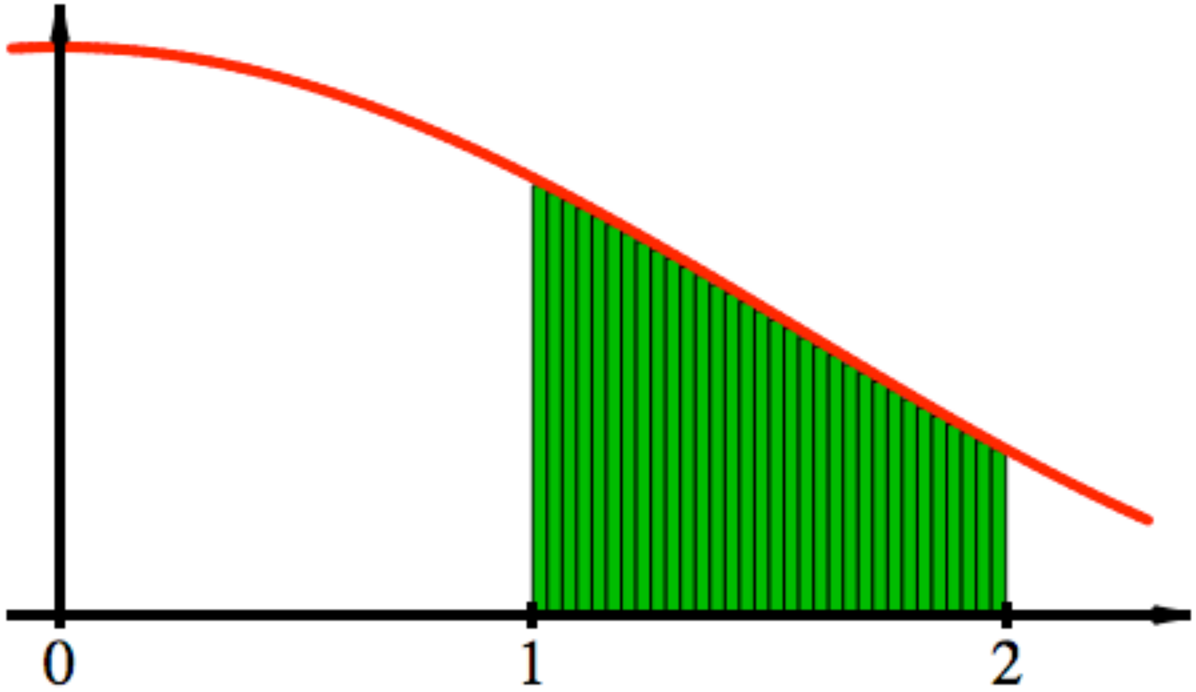


$$A \approx \sum_{k=1}^{8} f\left(1+\frac{k}{8}\right)\cdot\frac{1}{8}$$



$$A \approx \sum_{k=1}^{16} f\left(1 + \frac{k}{16}\right) \cdot \frac{1}{16}$$

Es wird immer weniger schlecht.



$$A \approx \sum_{k=1}^{32} f\left(1+\frac{k}{32}\right)\cdot\frac{1}{32}$$

Allgemein:

Schrittlänge = Δx

Schritte $= \frac{b-a}{\Delta x}$

$$A \approx \sum_{k=1}^{\frac{b-a}{\Delta x}} f\left(a + k\Delta x\right) \cdot \Delta x$$

$$A = \lim_{\Delta x \to 0} \sum_{k=1}^{\frac{b-a}{\Delta x}} f\left(a + k\Delta x\right) \cdot \Delta x \overset{\substack{\text{Schreib-}\\ \text{weise}\\ \downarrow}}{=} \int_a^b f\left(x\right) \mathrm{d}x$$

Idee von Leibniz

$$\int_a^b f(x)\,\mathrm{d}x$$

$\int$ wie $\int$umme

(∞ – viele Summanden)

dx ist ein ∞–kleiner Schritt

Idee von Leibniz

$$\int_a^b f(x)\,\mathrm{d}x$$

$\int$ wie $\int$umme

(∞ – viele Summanden)

dx ist ein ∞–kleiner Schritt

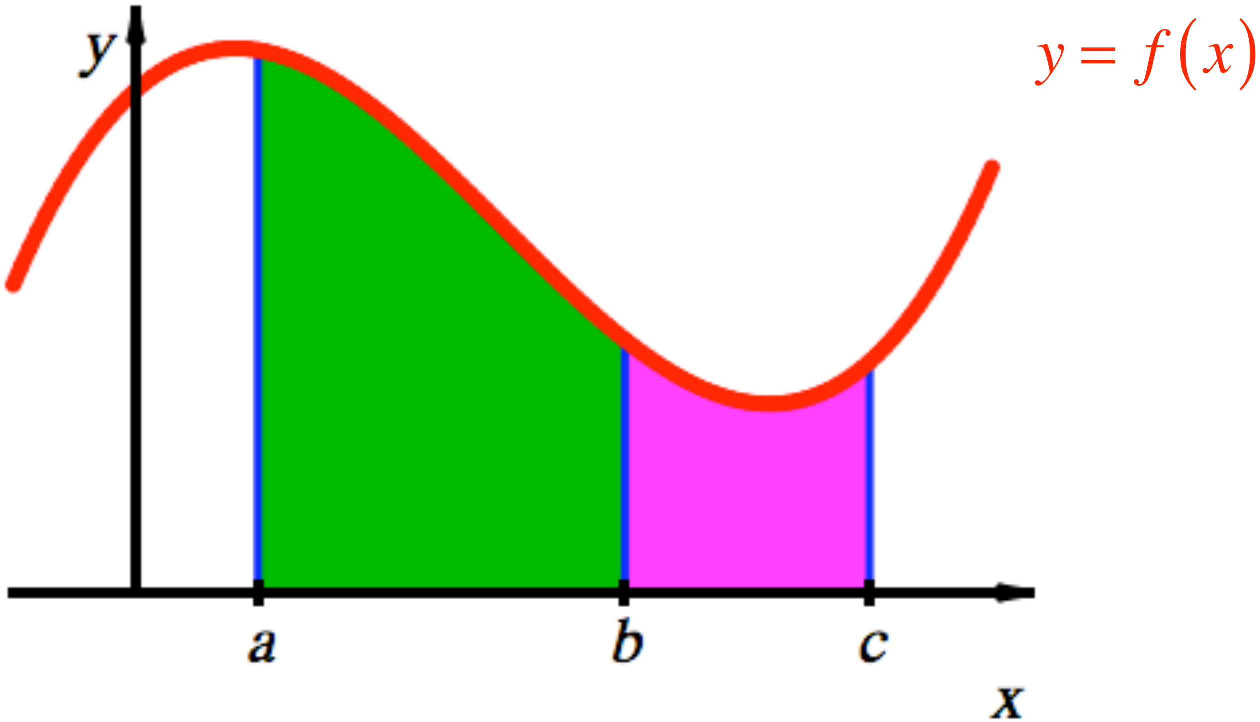
Sonderfälle

$$\int_a^a f(x)\,\mathrm{d}x = 0$$

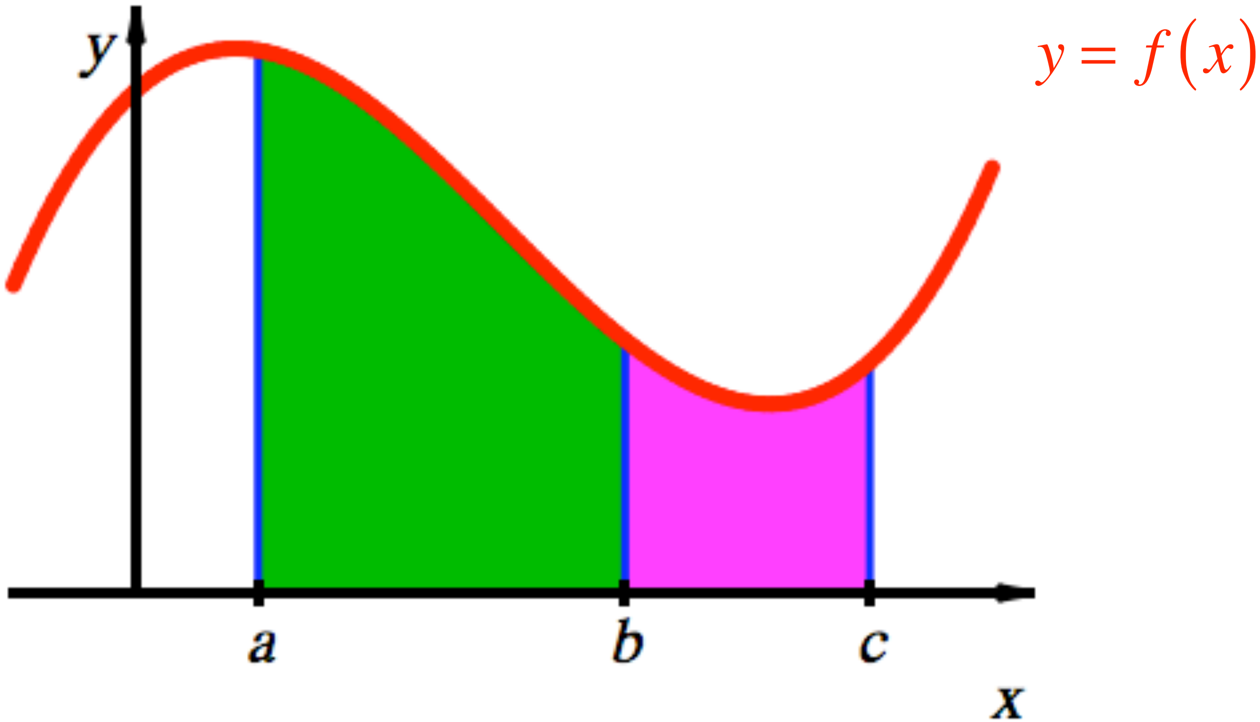
$$\int_a^b f(x)\,\mathrm{d}x = -\int_b^a f(x)\,\mathrm{d}x$$

warum?

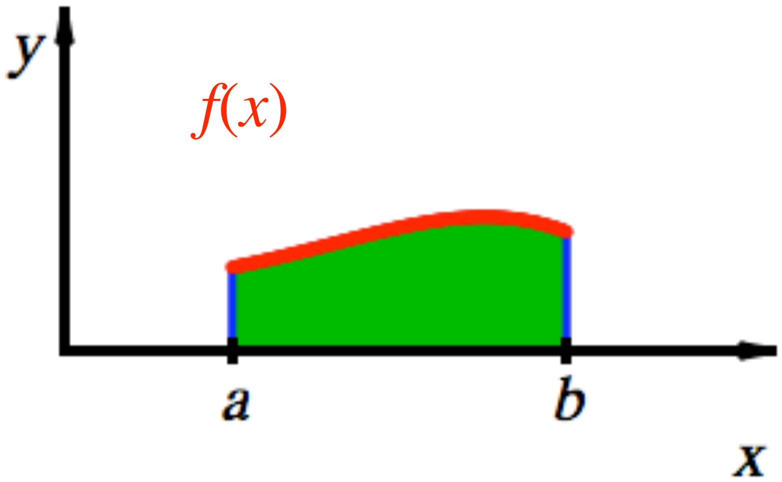
Aneinander

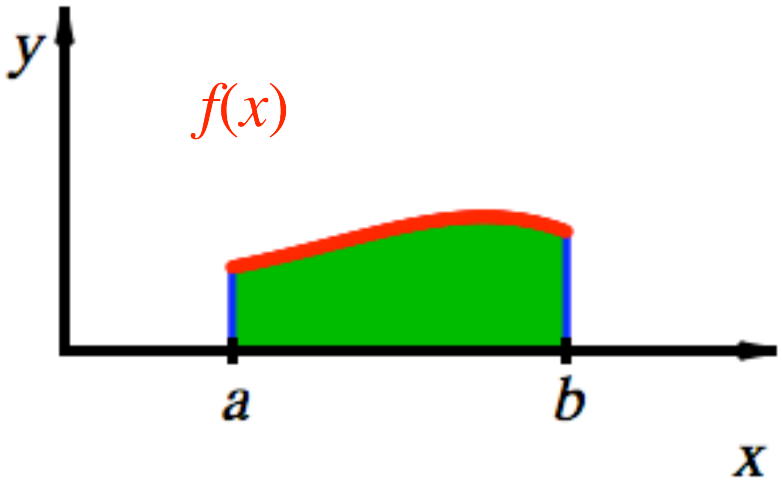


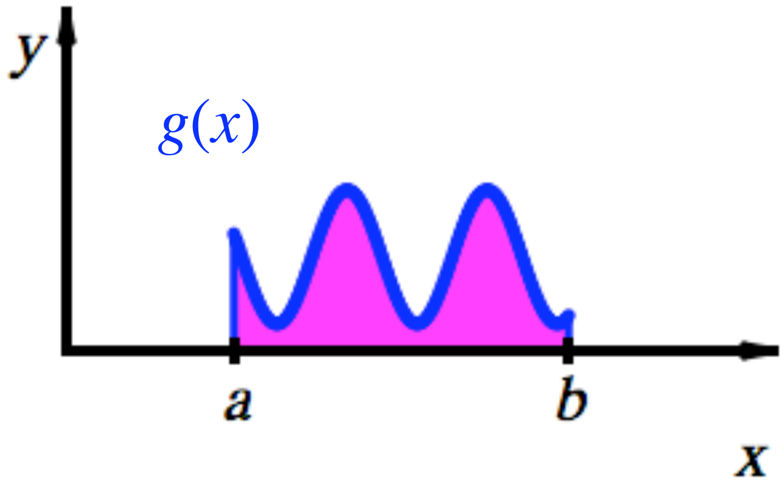
Aneinander

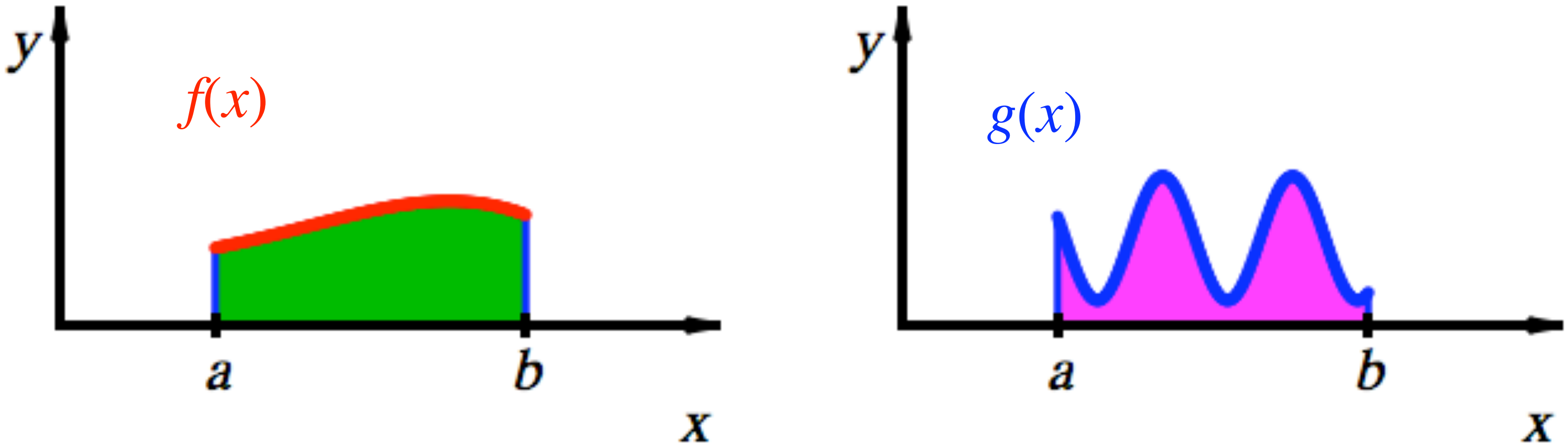


$$\int_a^b f(x)\,\mathrm{d}x + \int_b^c f(x)\,\mathrm{d}x = \int_a^c f(x)\,\mathrm{d}x$$

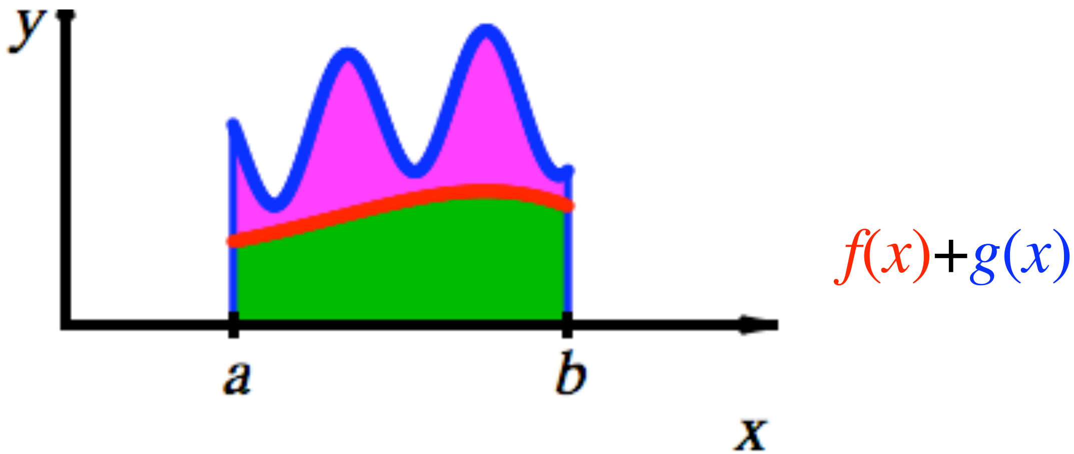

y
f(x)
a
b
x

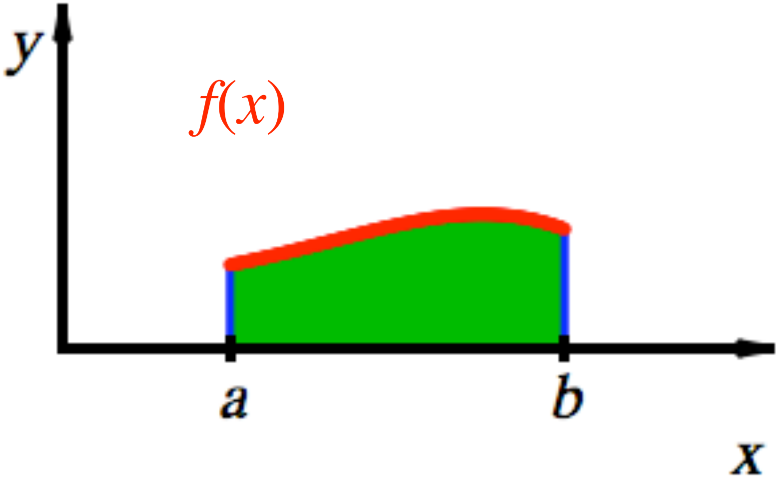

y
f(x)
a
b
x

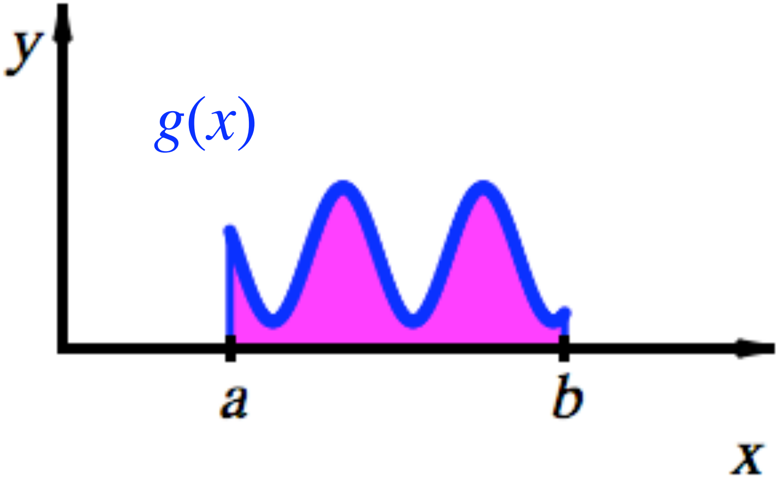

y
g(x)
a
b
x



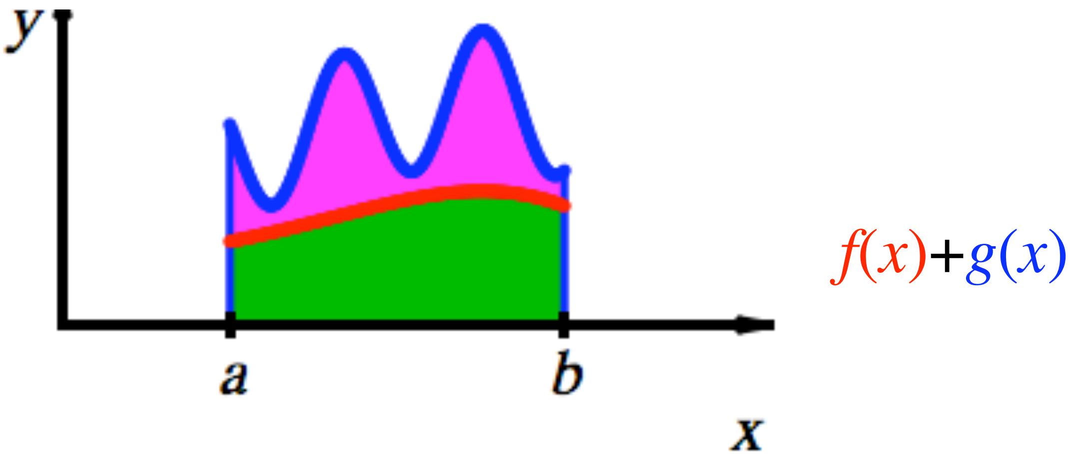
Aufeinander



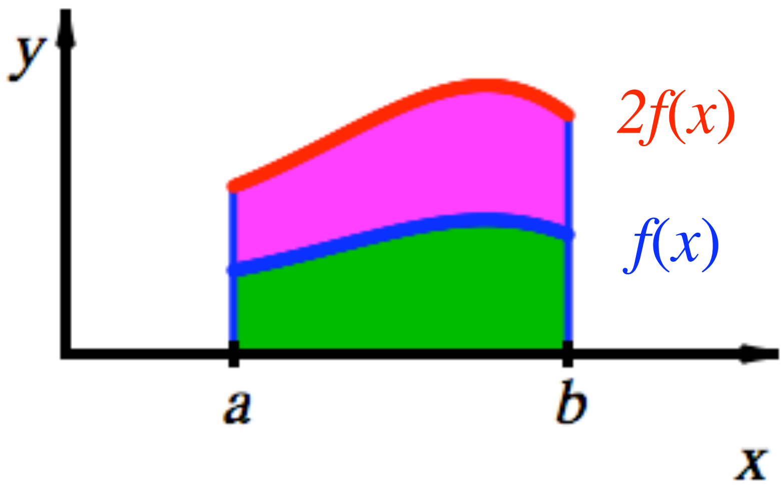




Aufeinander



$$\int_a^b \left(f(x)+g(x)\right)\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x + \int_a^b g(x)\,\mathrm{d}x$$



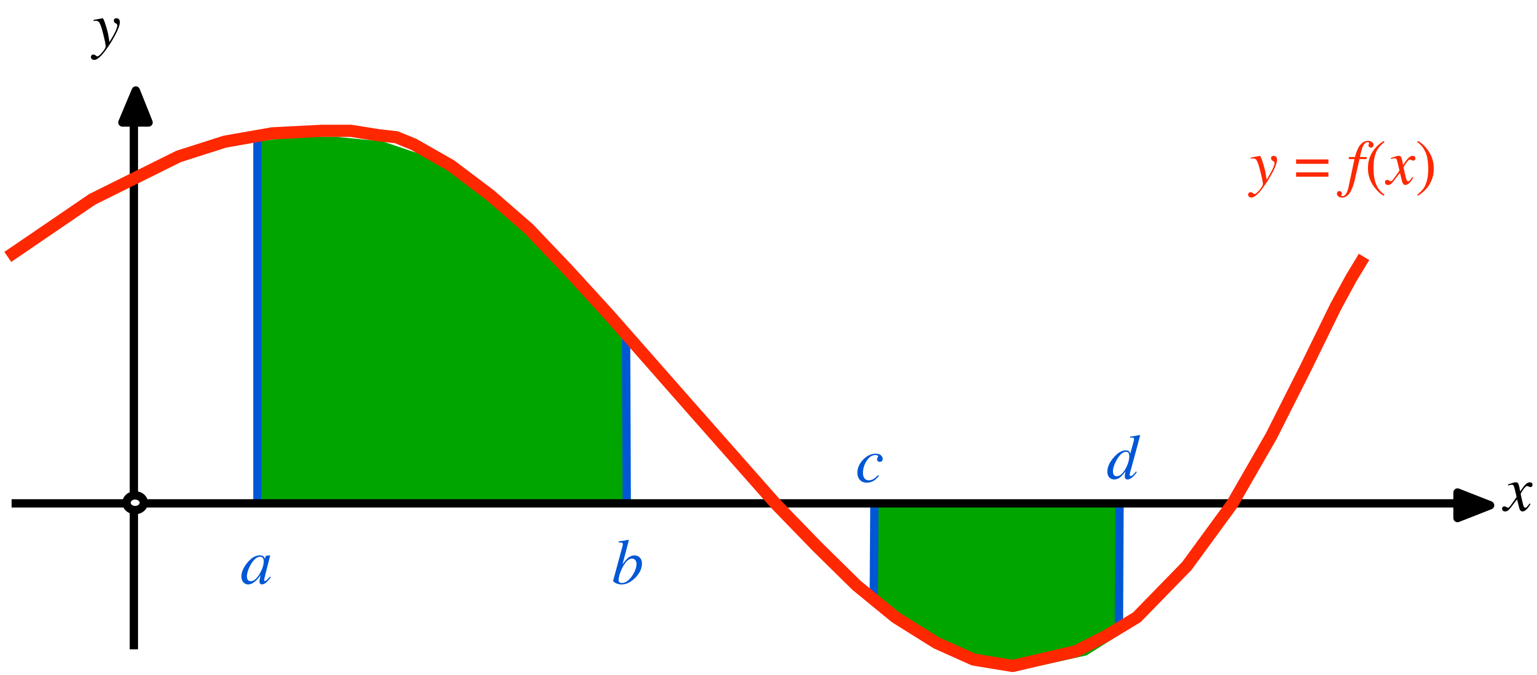
verdoppeln

ver-λ-fachen

$$\int_a^b \lambda f(x)\,\mathrm{d}x = \lambda \int_a^b f(x)\,\mathrm{d}x$$

Faktor λ herausnehmen (ausklammern)

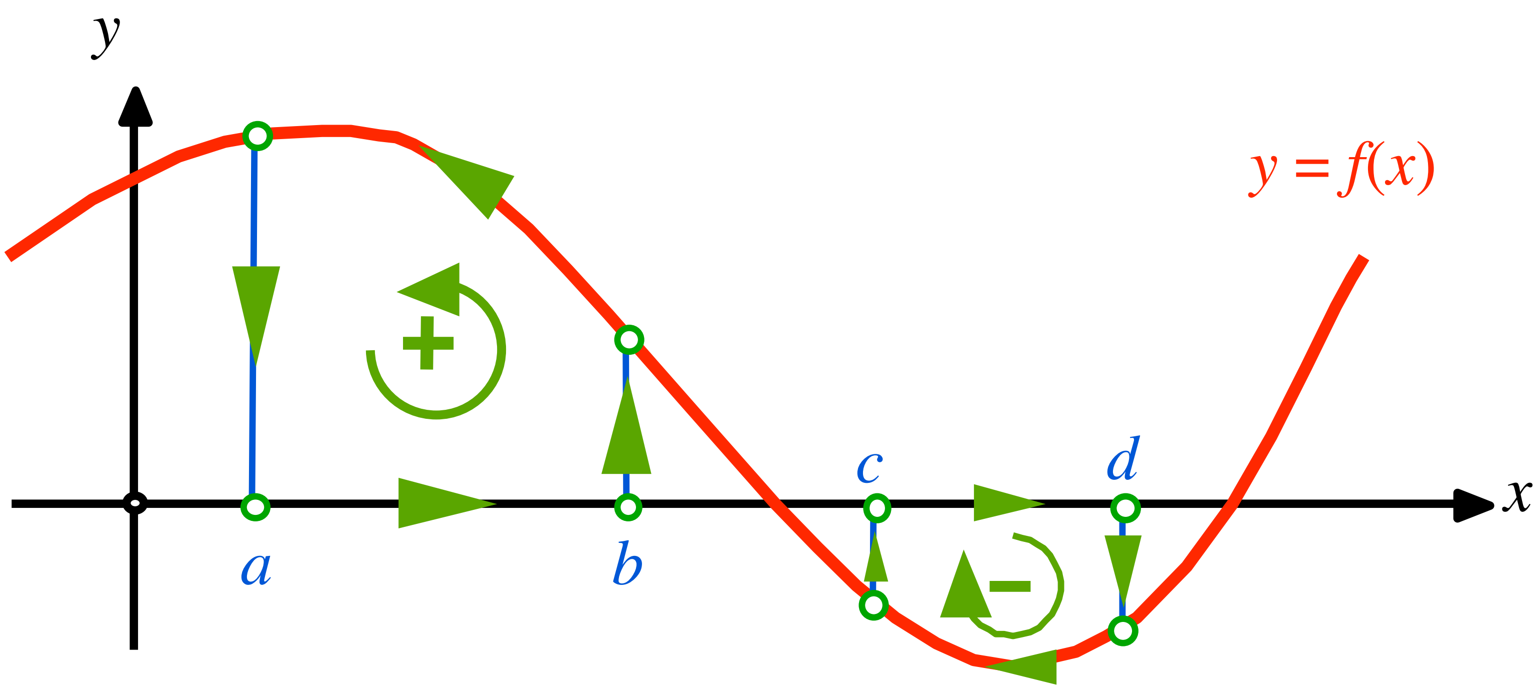
„Taucher“



$\int_a^b f(x)\,\mathrm{d}x$ positiv

$\int_c^d f(x)\,\mathrm{d}x$ negativ

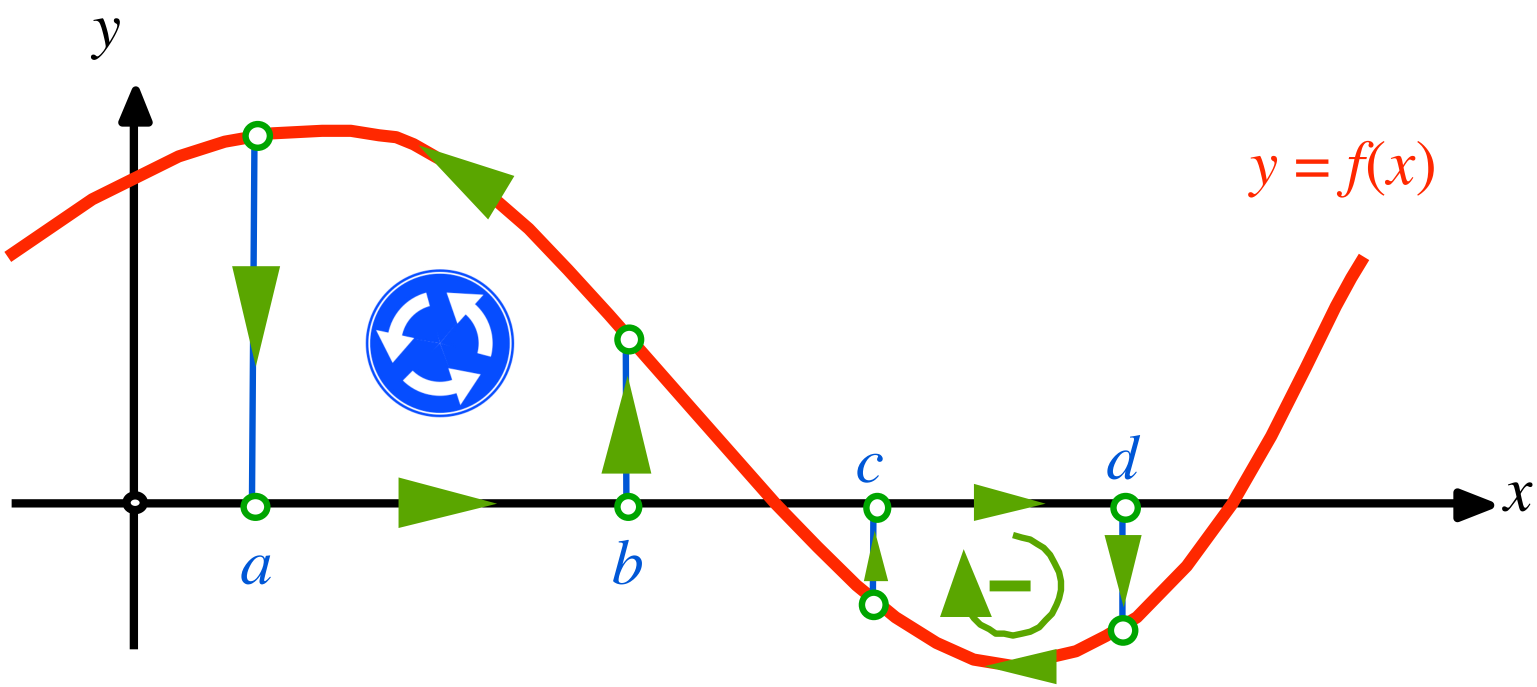
Orientierter Flächeninhalt



$$\int_a^b f(x)\,\mathrm{d}x \quad \text{positiv}$$

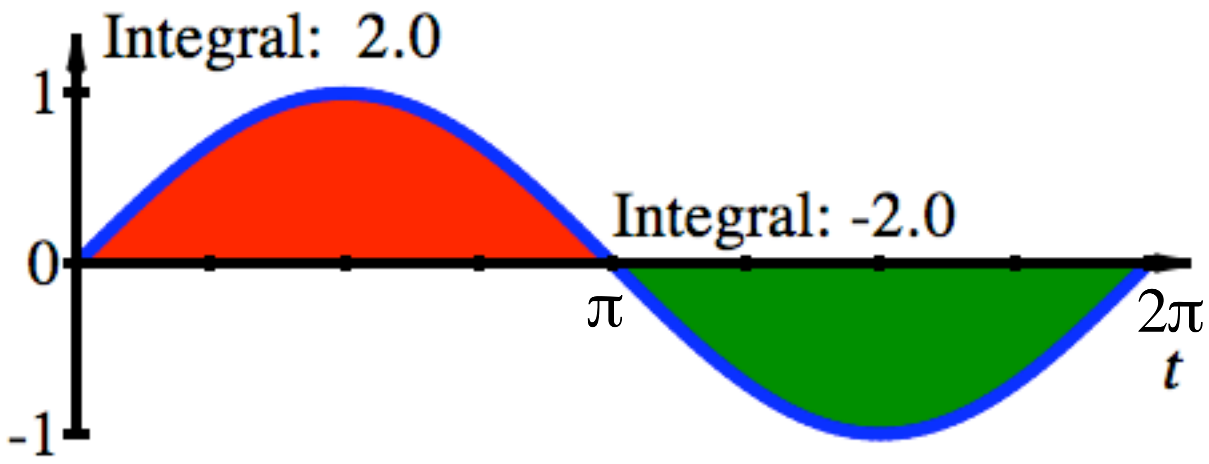
$$\int_c^d f(x)\,\mathrm{d}x \quad \text{negativ}$$

Orientierter Flächeninhalt



$$\int_a^b f(x)\,\mathrm{d}x \quad \text{positiv}$$

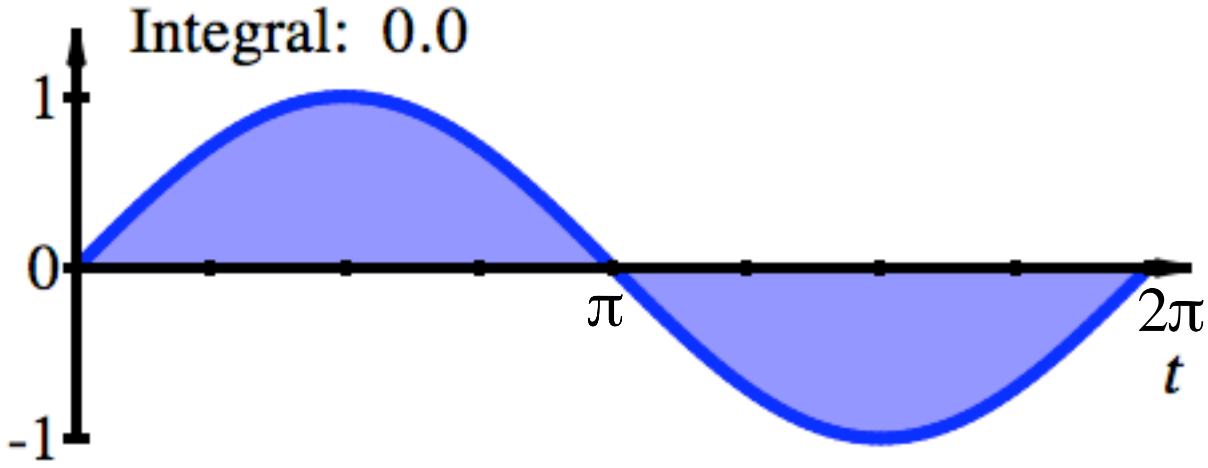
$$\int_c^d f(x)\,\mathrm{d}x \quad \text{negativ}$$



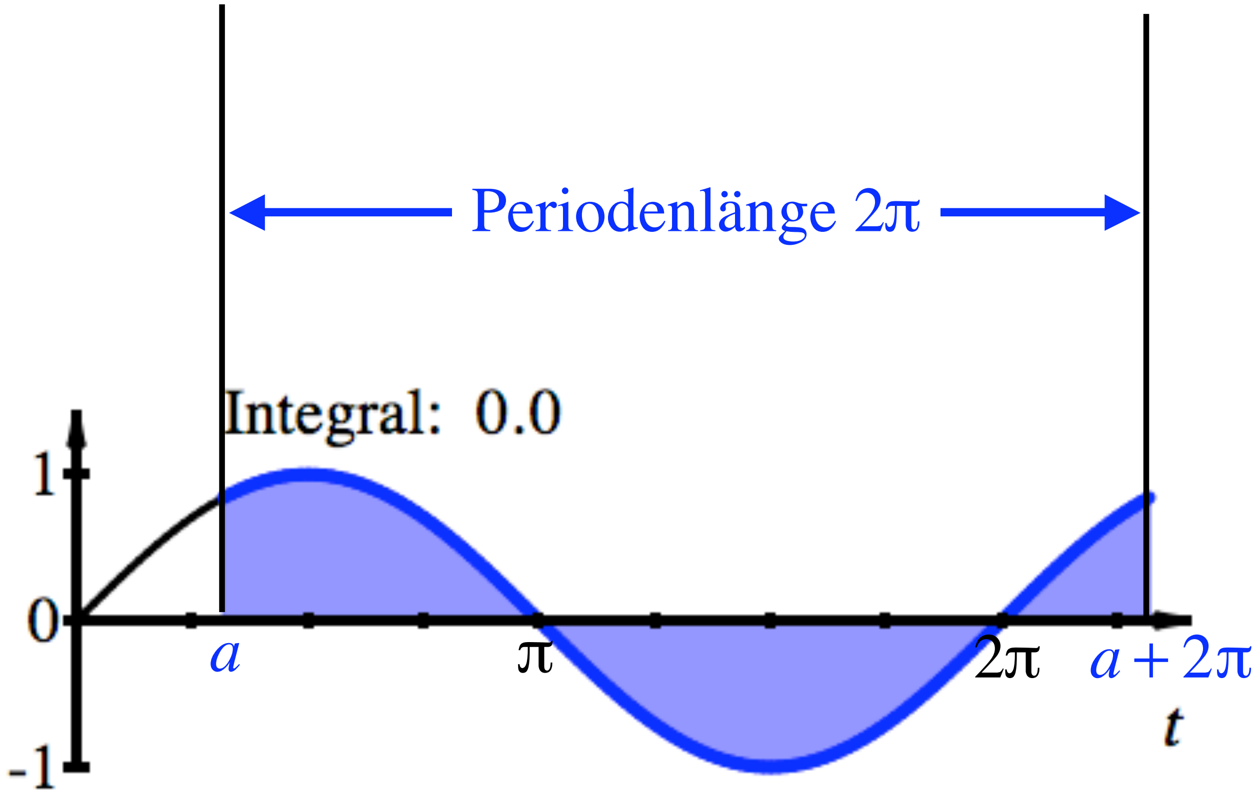
$$\int_0^{\pi} \sin(t)\,\mathrm{d}t = +2$$

$$\int_{\pi}^{2\pi} \sin(t)\,\mathrm{d}t = -2$$

$$\int_0^{2\pi} \sin(t)\,\mathrm{d}t = ?$$



$$\int_0^{2\pi} \sin(t)\,\mathrm{d}t = 0$$



$$\int_{a}^{a+2\pi} \sin(t)\,\mathrm{d}t = 0$$

Unterschied?

$$\left| \int_0^{2\pi} \sin(t)\,\mathrm{d}t \right| =$$

$$\int_0^{2\pi} \left|\sin(t)\right|\,\mathrm{d}t =$$

Unterschied?

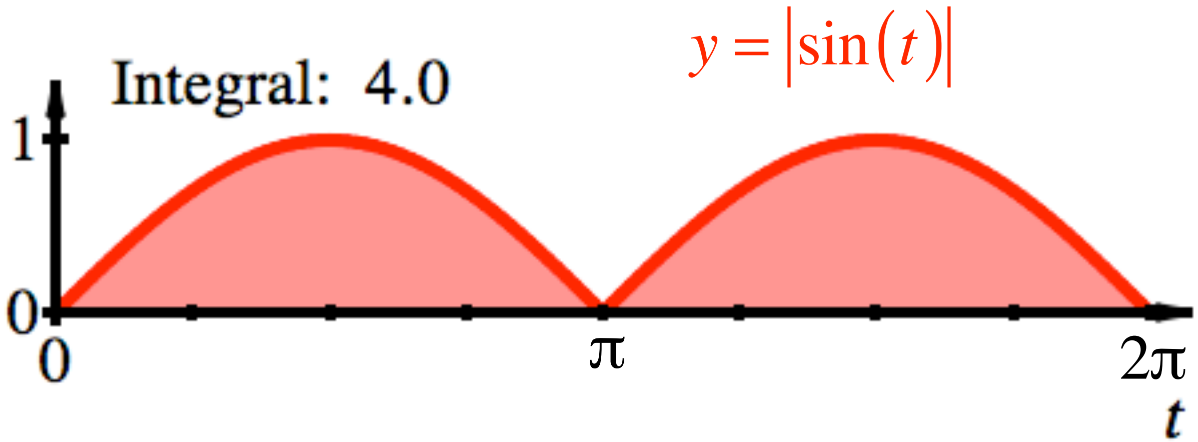
$$\left|\int_0^{2\pi} \sin(t)\,\mathrm{d}t\right| = |0| = 0$$

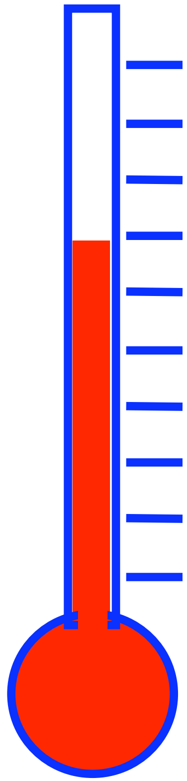
$$\int_0^{2\pi} |\sin(t)|\,\mathrm{d}t =$$

Unterschied?

$$\left|\int_0^{2\pi} \sin(t)\,\mathrm{d}t\right| = |0| = 0$$

$$\int_0^{2\pi} |\sin(t)|\,\mathrm{d}t = 4$$

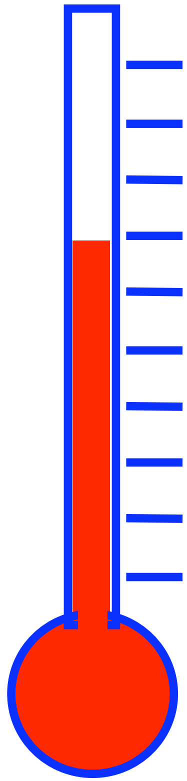




Wie warm ist es heute?

Momentane Temperatur?

Mittlere Tagestemperatur?



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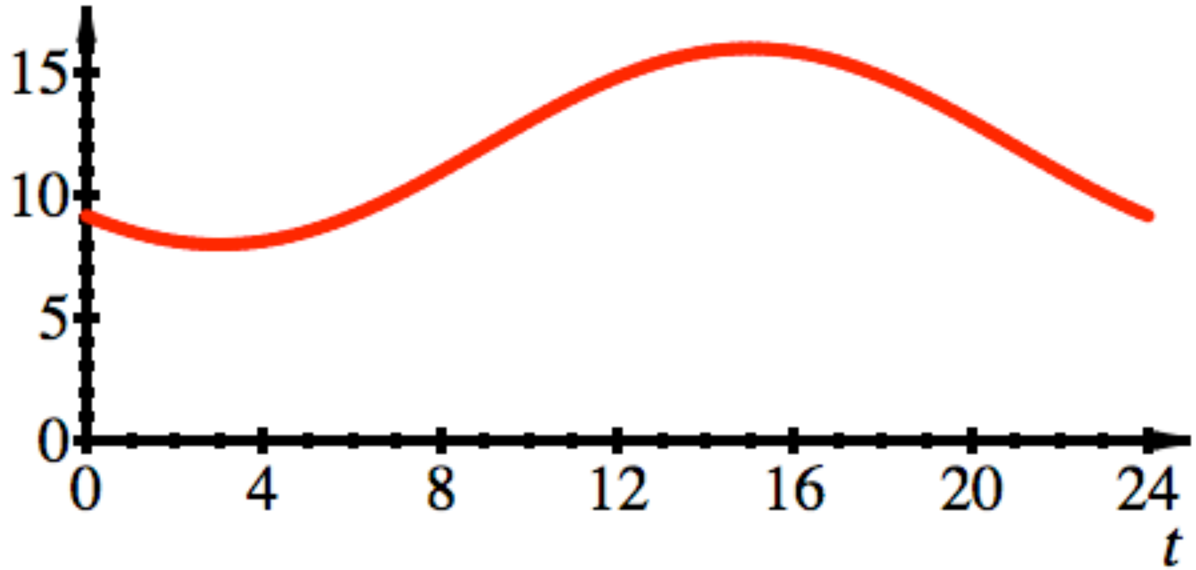
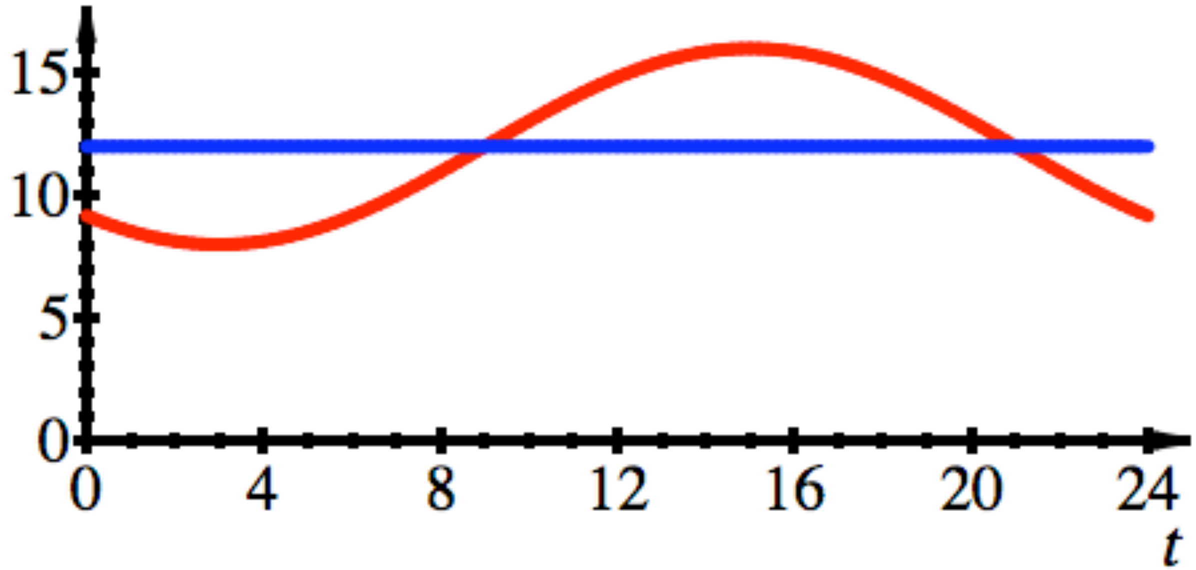


Momentane Temperatur
variabel

Momentane Temperatur?

Mittlere Tagestemperatur?

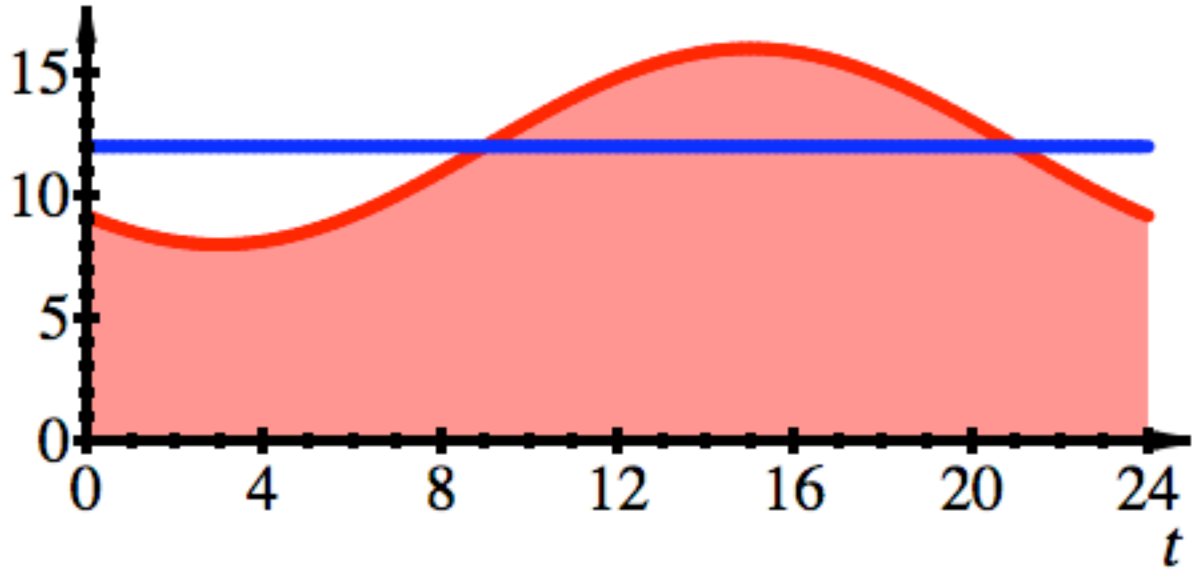


Momentane Temperatur
variabel

Mittlere Temperatur
ca. 12°

Momentane Temperatur?

Mittlere Tagestemperatur?

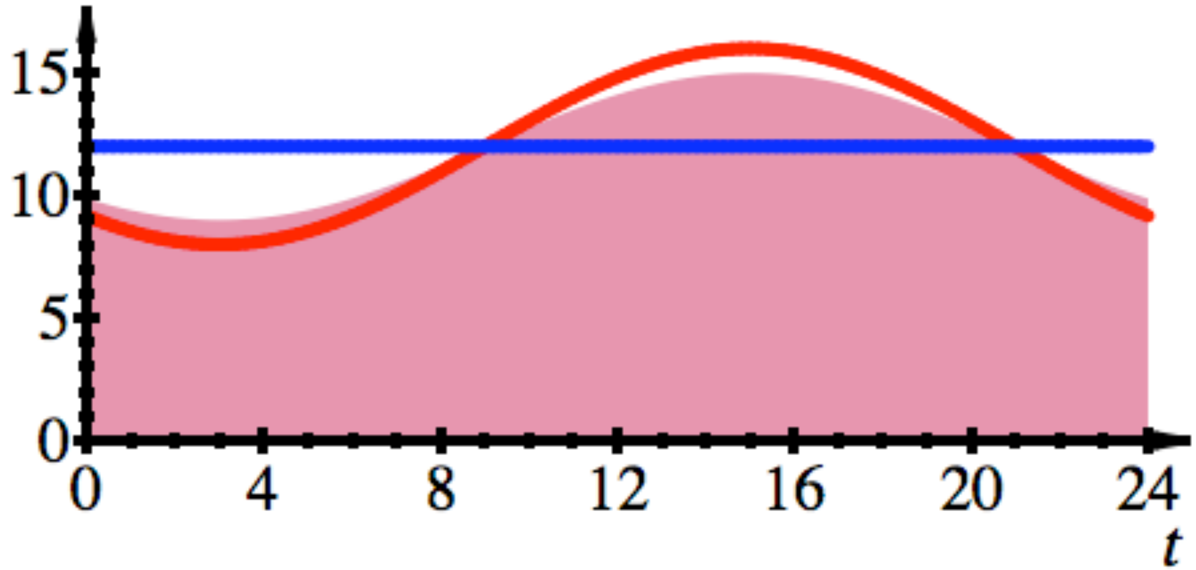


Momentane Temperatur
variabel

Mittlere Temperatur
ca. 12°

Momentane Temperatur?

Mittlere Tagestemperatur?

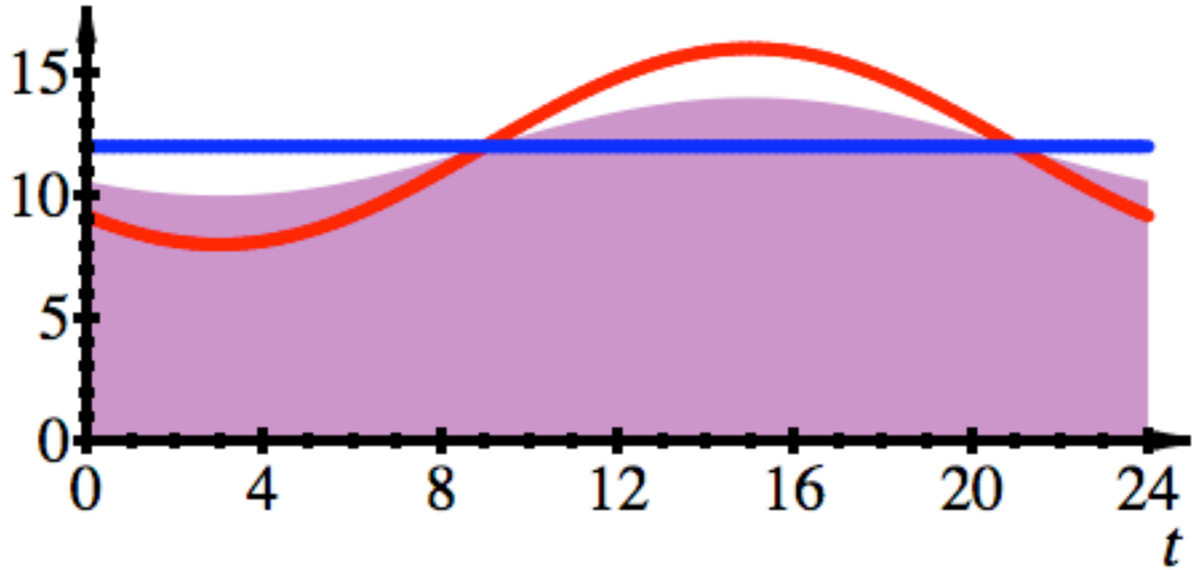


Momentane Temperatur
variabel

Mittlere Temperatur
ca. 12°

Momentane Temperatur?

Mittlere Tagestemperatur?

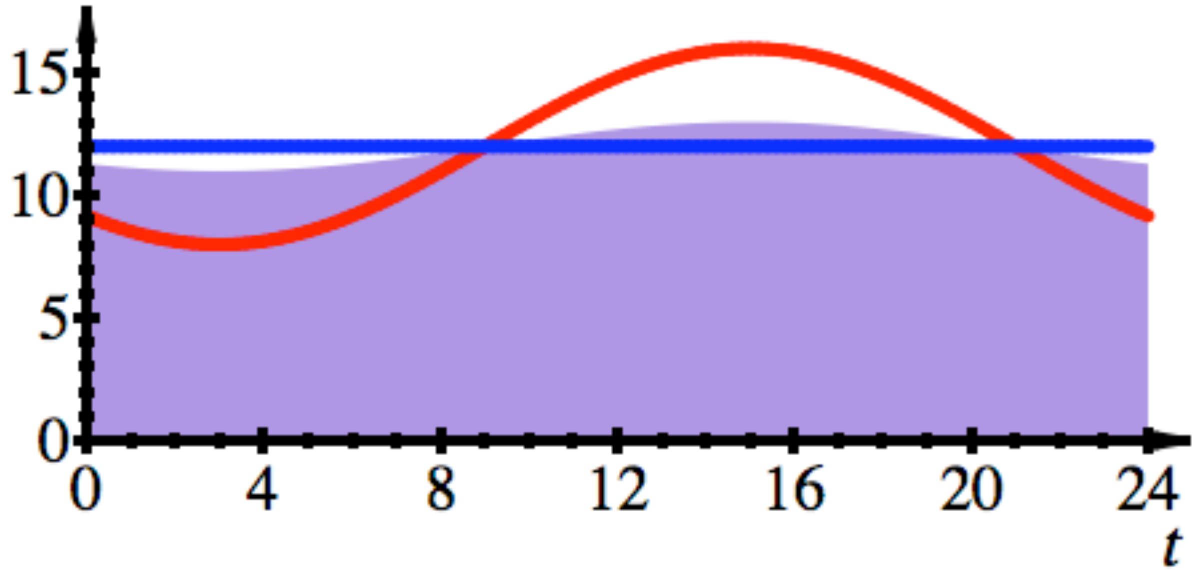


Momentane Temperatur variabel

Mittlere Temperatur ca. 12°

Momentane Temperatur?

Mittlere Tagestemperatur?

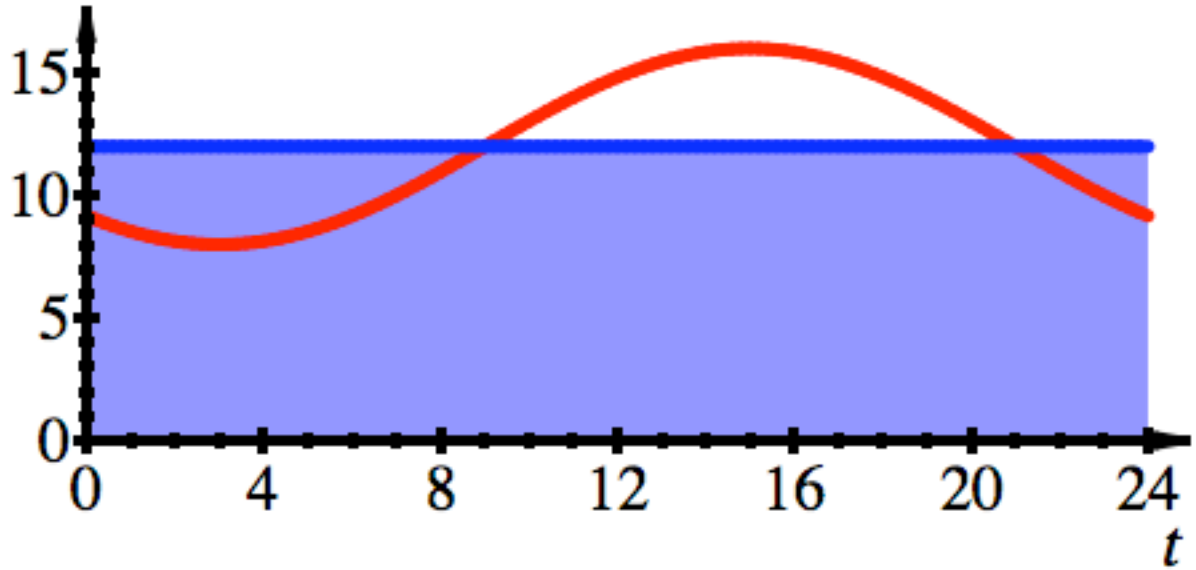


Momentane Temperatur
variabel

Mittlere Temperatur
ca. 12°

Momentane Temperatur?

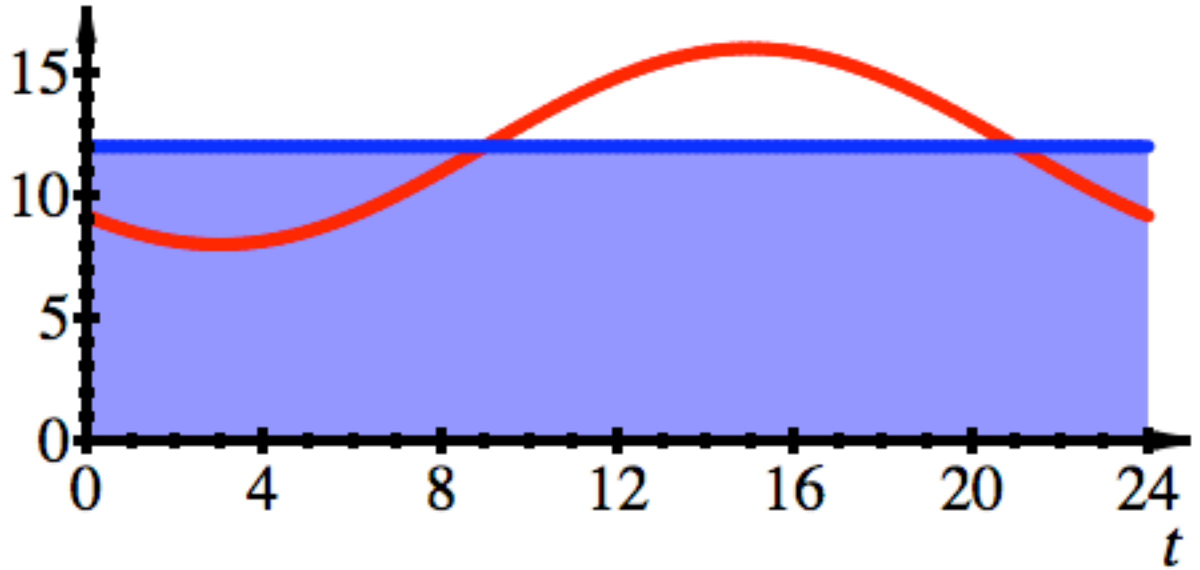
Mittlere Tagestemperatur?



Momentane Temperatur
variabel

Mittlere Temperatur
ca. 12°

$$\text{Integralmittelwert} = \frac{\int_a^b f(x)\,\mathrm{d}x}{b-a}$$



Momentane Temperatur
variabel

Mittlere Temperatur
ca. 12°

$$\text{Integralmittelwert} = \frac{\int_a^b f(x)\,\mathrm{d}x}{b-a}$$

$$\text{Durchschnitt} = \frac{\text{Summe}}{\text{Anzahl}}$$

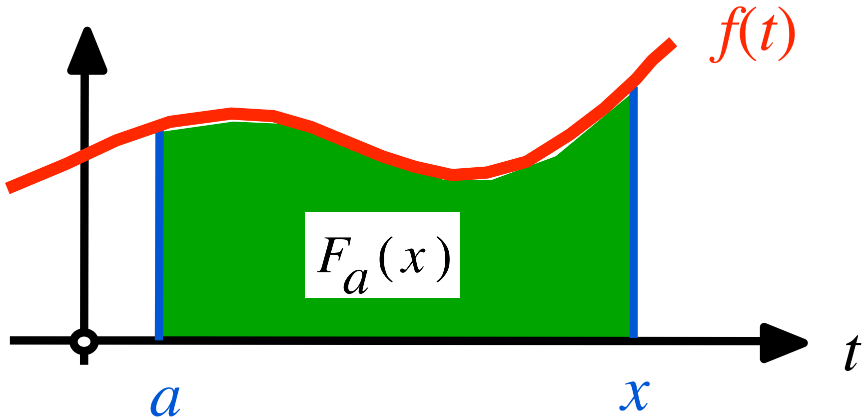
Hauptsatz der Differenzial- und Integralrechung

Spiel in 9 Szenen

Am Schluss wissen Sie soviel wie vorher,
aber Sie wissen es besser.

Szene 1

$$F_a(x) = \int_a^x f(t)\,\mathrm{d}t$$



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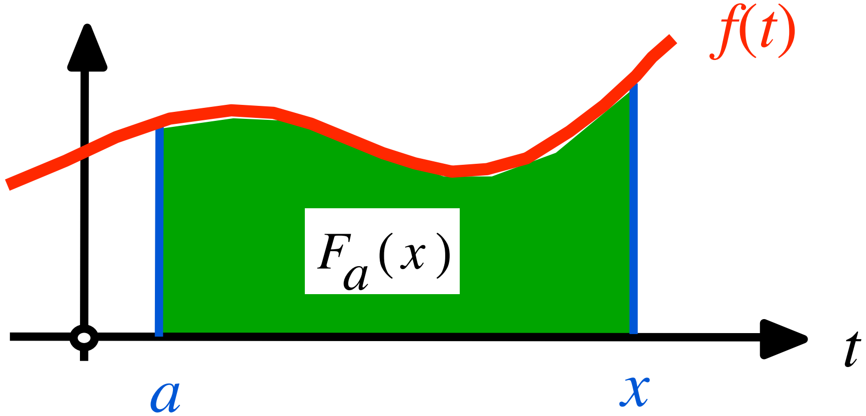
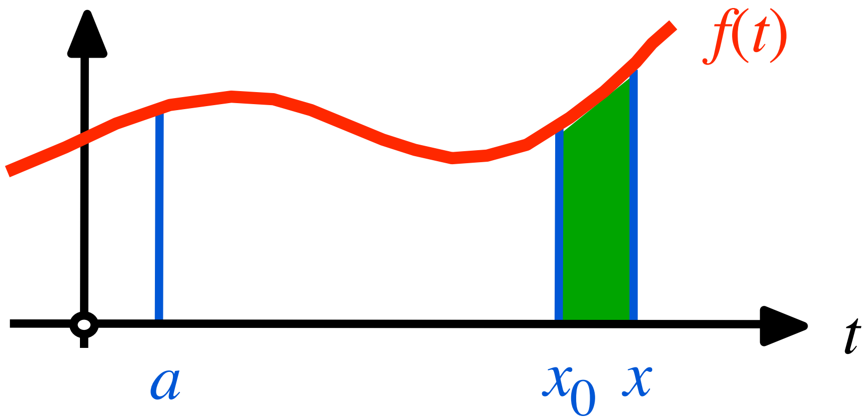


Szene 2

$$F_a(x) - F_a(x_0) = \int_{x_0}^{x} f(t)\,\mathrm{d}t$$



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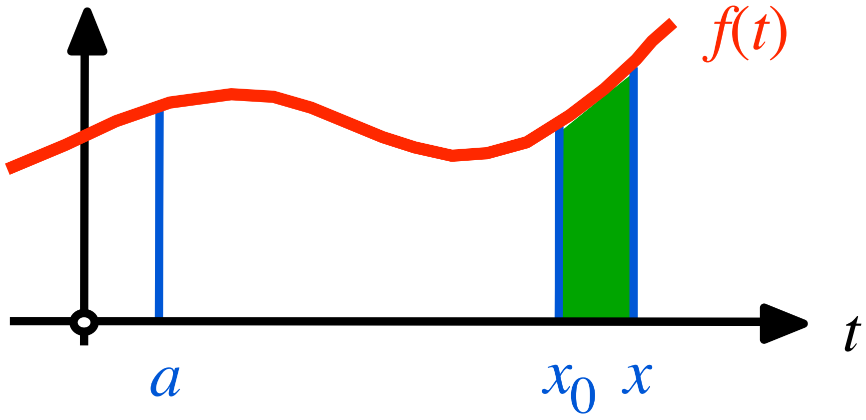

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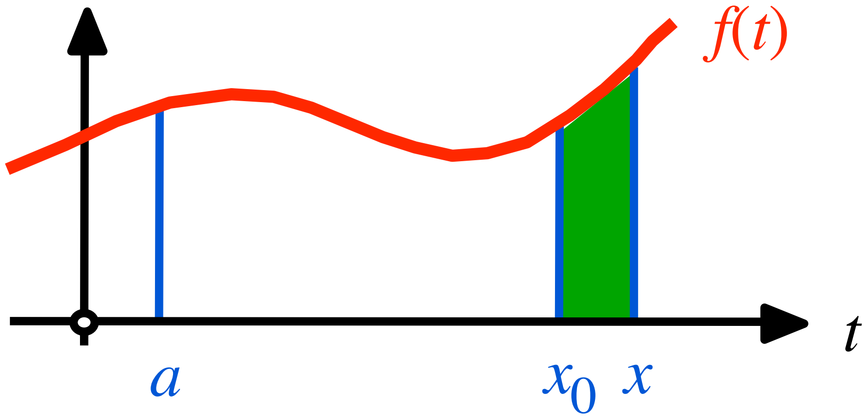
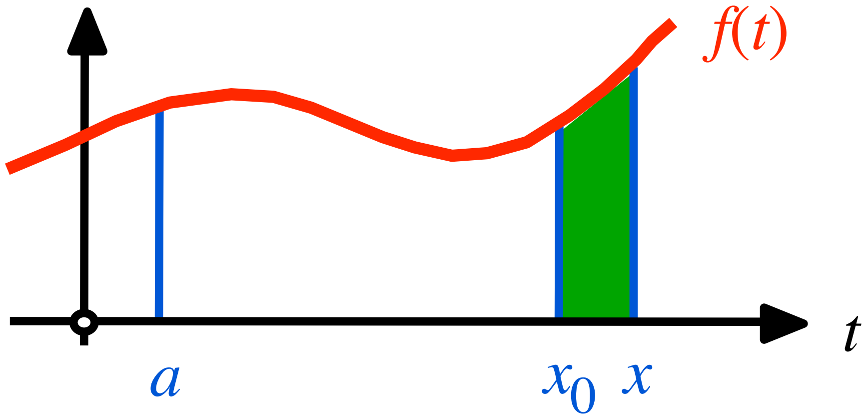


Szene 3

$$\frac{F_a(x) - F_a(x_0)}{x - x_0} \approx f(x_0)$$

Szene 2

$$F_a(x) - F_a(x_0) = \int_{x_0}^{x} f(t)\,\mathrm{d}t$$



Szene 3

$$\frac{F_a(x) - F_a(x_0)}{x - x_0} \approx f(x_0)$$

Szene 4

$$\lim_{x \to x_0} \frac{F_a(x) - F_a(x_0)}{x - x_0} = f(x_0) \quad \Rightarrow \quad F_a'(x_0) = f(x_0)$$

Szene 4

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Szene 4 $$\lim_{x\to x_0} \frac{F_a(x)-F_a(x_0)}{x-x_0} = f(x_0) \quad \Rightarrow \quad F_a'(x_0) = f(x_0)$$

Szene 5 F_a ist eine (spezielle) Stammfunktion von f

Es ist $F_a(a) = 0$

Szene 4 $\lim\limits_{x \to x_0} \frac{F_a(x) - F_a(x_0)}{x - x_0} = f(x_0) \quad \Rightarrow \quad F_a'(x_0) = f(x_0)$

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Szene 6 beliebige Stammfunktion von f

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Szene 6 beliebige Stammfunktion von f

$$F(x) = F_a(x) + C$$

Szene 7

$$\int_a^b f(t)\,\mathrm{d}t = F_a(b)$$

Szene 6 beliebige Stammfunktion von f

$$F(x) = F_a(x) + C$$

Szene 7

$$\int_a^b f(t)\,\mathrm{d}t = F_a(b)$$

Szene 8

$$\underbrace{\underset{\uparrow}{F}(b) - \underset{\uparrow}{F}(a)}_{\text{beliebige Stammfunktion}} = \left(F_a(b) + C\right) - \left(\underbrace{F_a(a)}_{=0} + C\right) = F_a(b)$$

Szene 7

$$\int_a^b f(t)\,\mathrm{d}t = F_a(b)$$

Szene 8

$$\underbrace{\underset{\uparrow}{F}(b) - \underset{\uparrow}{F}(a)}_{\substack{\text{beliebige}\\ \text{Stammfunktion}}} = \big(F_a(b) + C\big) - \big(\underbrace{F_a(a)}_{=0} + C\big) = F_a(b)$$

Szene 9

$$\int_a^b f(t)\,\mathrm{d}t = \underbrace{\underset{\uparrow}{F}(b) - \underset{\uparrow}{F}(a)}_{\substack{\text{beliebige}\\ \text{Stammfunktion}}}$$

Hauptsatz

$$\int_a^b f(t)\,\mathrm{d}t = F(b) - F(a)$$

Beliebige Stamm-funktion von f

$$F'(t) = f(t)$$

Ableiten ist die Umkehrung des Integrierens.

Schreibweise

$$\int_a^b f(x)\,\mathrm{d}x = F(x)\Big|_a^b$$

Schreibweise

$$\int_a^b f(x)\,\mathrm{d}x = F(x)\Big|_a^b$$

Beispiel:

$$\int_1^3 x^2\,\mathrm{d}x = \tfrac{1}{3}x^3\Big|_1^3$$

$$= \tfrac{1}{3}3^3 - \tfrac{1}{3}1^3$$

$$= 9 - \tfrac{1}{3} = \frac{26}{3}$$

Probleme an den Grenzen

„uneigentliche“ Integrale

Tipp:
Grenze von der sicheren Seite her anschleichen.

Probleme an den Grenzen

Beispiele

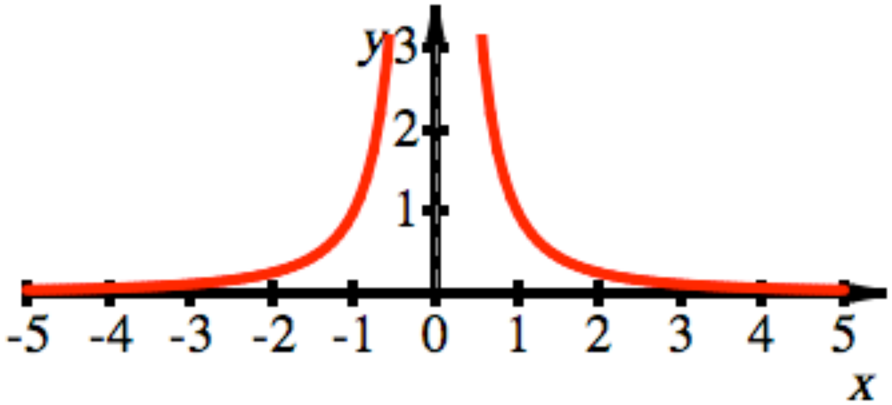
$$f(x) = \frac{1}{x^2}$$

$$f(x) = \frac{1}{\sqrt{x}}$$

$$f(x) = \frac{1}{x}$$

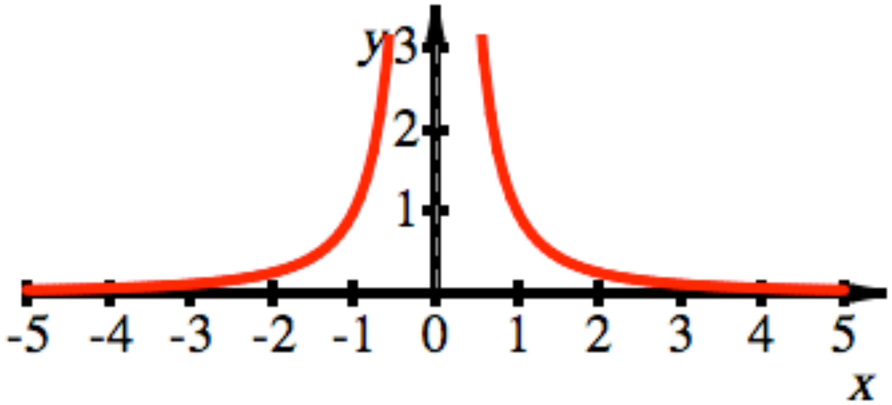
Problematische Grenzen: 0, $+\infty$, $-\infty$

Beispiel: $f(x) = \frac{1}{x^2}$



Heiße Grenze $+\infty$

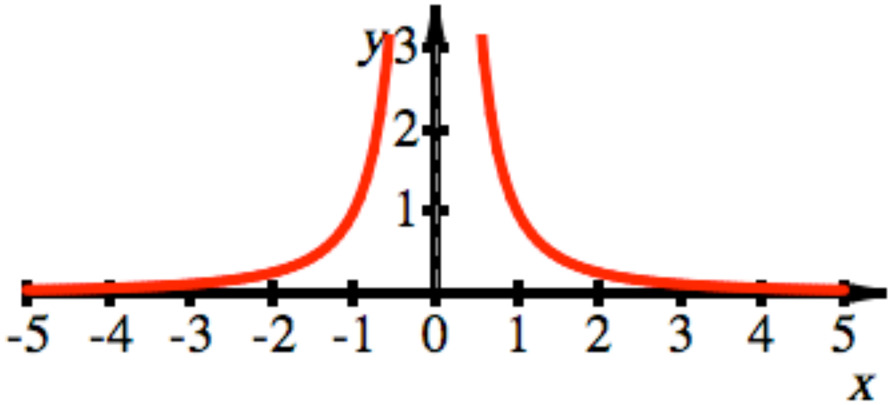
Beispiel: $f(x) = \frac{1}{x^2}$



Heiße Grenze $+\infty$

$$f(x) = \frac{1}{x^2} \quad \Rightarrow \quad \int \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{x} + C$$

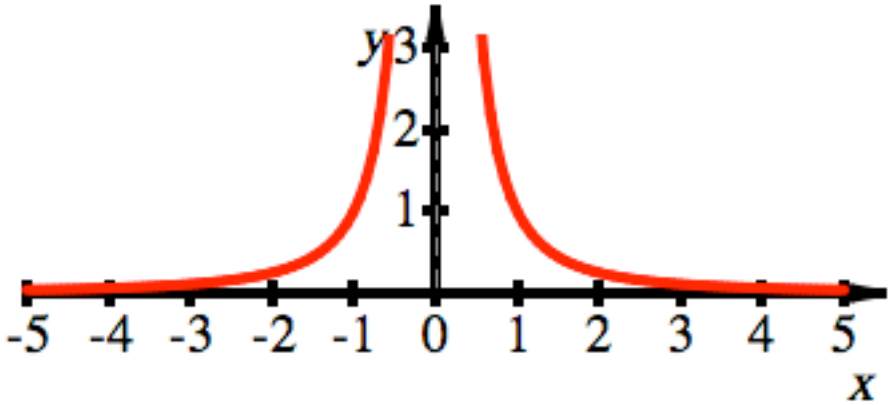
Beispiel: $f(x) = \frac{1}{x^2}$



Heiße Grenze $+\infty$

$$f(x) = \frac{1}{x^2} \quad \Rightarrow \quad \int \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{x} + C \quad \Rightarrow \quad \int_1^r \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{r} + \frac{1}{1}$$

Beispiel: $f(x)=\frac{1}{x^2}$



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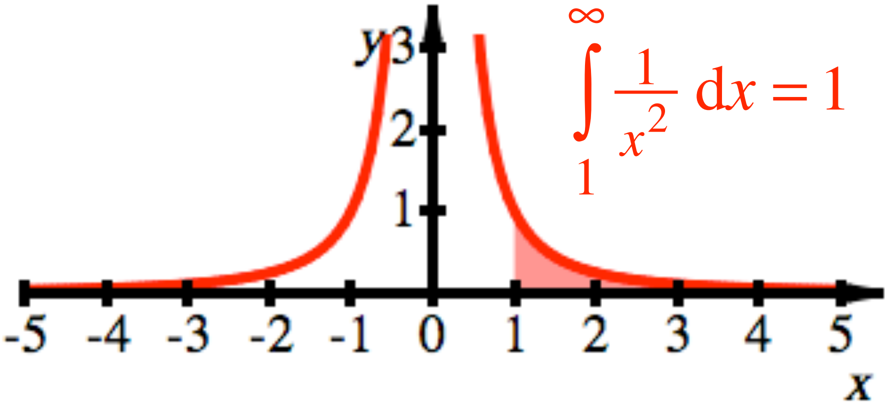
$$f(x)=\frac{1}{x^2} \quad \Rightarrow \quad \int \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{x}+C \quad \Rightarrow \quad \int_1^r \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{r}+\frac{1}{1}$$

$$\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x = \lim_{r\to\infty}\left(\int_1^r \frac{1}{x^2}\,\mathrm{d}x\right) = \lim_{r\to\infty}\left(-\frac{1}{r}+1\right)=1$$

Beispiel: $f(x) = \frac{1}{x^2}$



$$\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x = 1$$

Heiße Grenze $+\infty$

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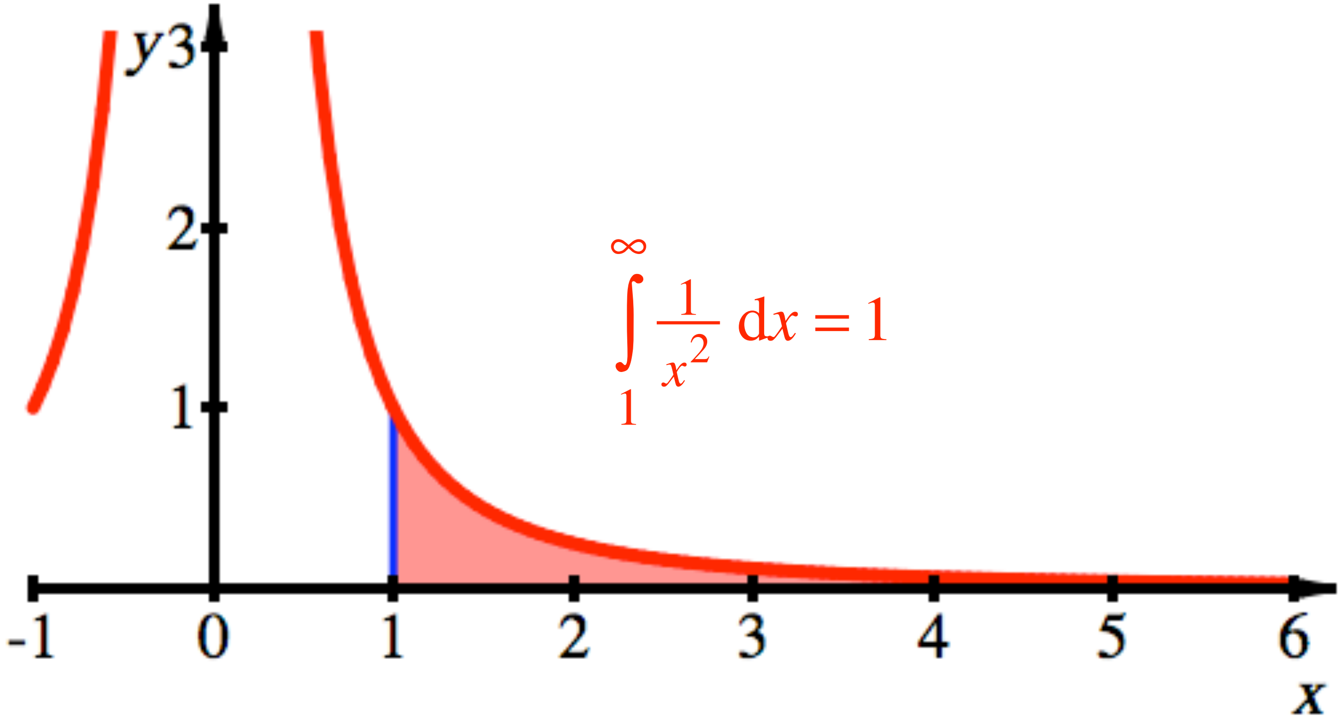
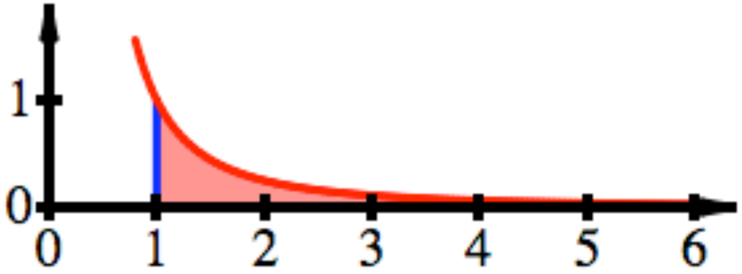
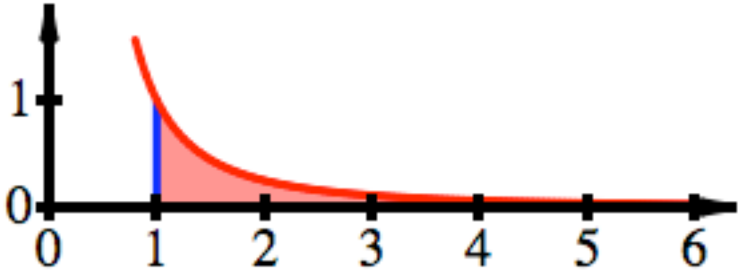


$$\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x = 1$$



$$\int_1^{\infty} \frac{1}{x^2}\, dx = 1$$



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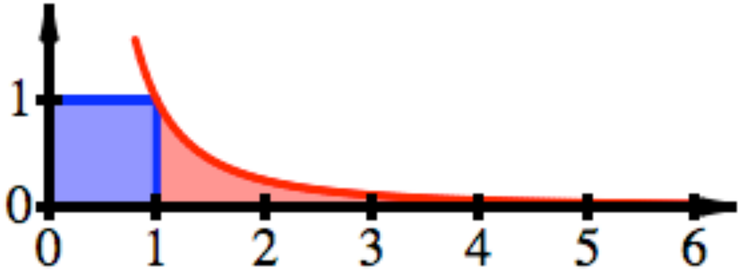
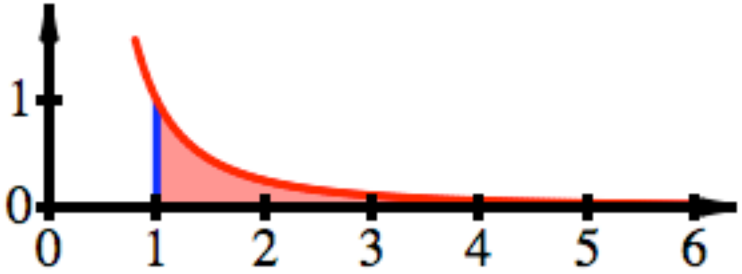


Einheitsquadrat addieren, gibt 2



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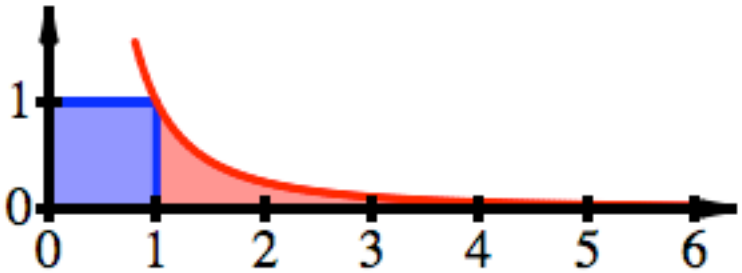

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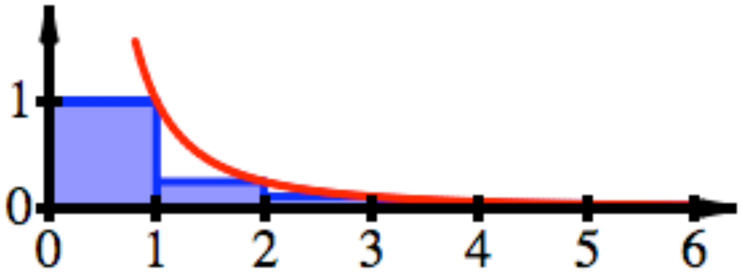
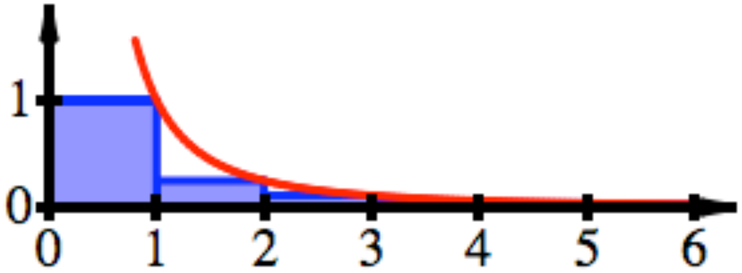


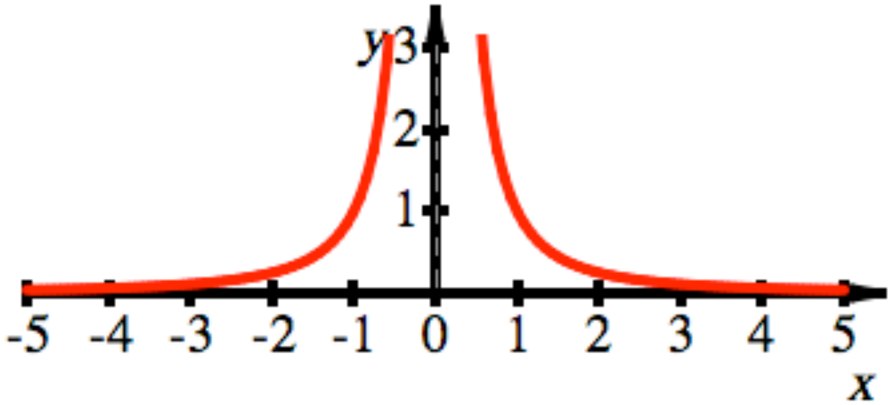
$$1 + \frac{1}{4} + \frac{1}{9} + \cdots = \sum_{k=1}^{\infty} \frac{1}{k^2} < 2$$



$$1+\frac{1}{4}+\frac{1}{9}+\cdots=\sum_{k=1}^{\infty}\frac{1}{k^2}<2$$

EULER: $$1+\frac{1}{4}+\frac{1}{9}+\cdots=\sum_{k=1}^{\infty}\frac{1}{k^2}=\frac{\pi^2}{6}\approx 1.645$$

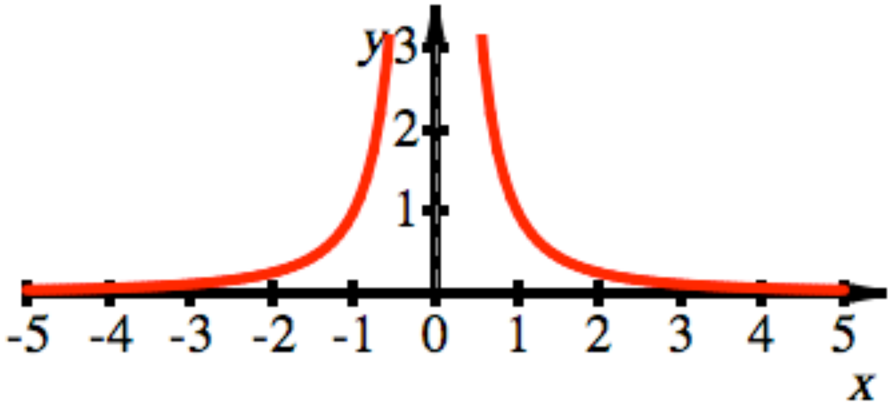
Beispiel: $f(x) = \frac{1}{x^2}$



Heiße Grenze 0

$$f(x) = \frac{1}{x^2} \quad \Rightarrow \quad \int \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{x} + C$$

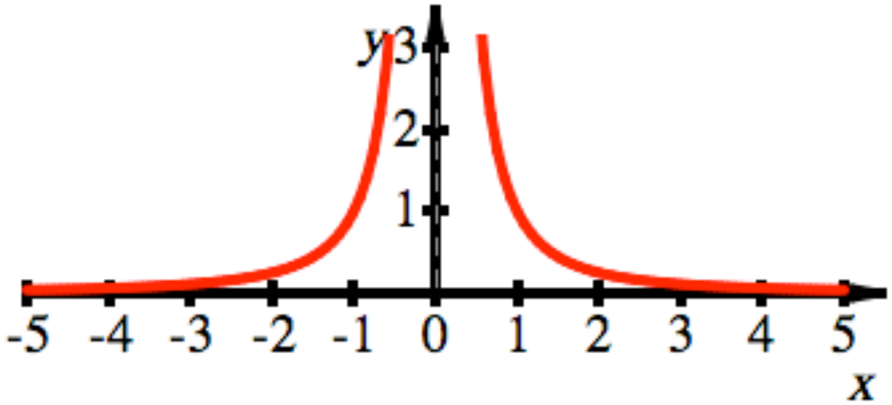
Beispiel: $f(x)=\frac{1}{x^2}$



Heiße Grenze 0

$$f(x)=\frac{1}{x^2} \quad\Rightarrow\quad \int \frac{1}{x^2}\,\mathrm{d}x=-\frac{1}{x}+C \quad\Rightarrow\quad \int_s^1 \frac{1}{x^2}\,\mathrm{d}x=-\frac{1}{1}+\frac{1}{s}$$

Beispiel: $f(x) = \frac{1}{x^2}$

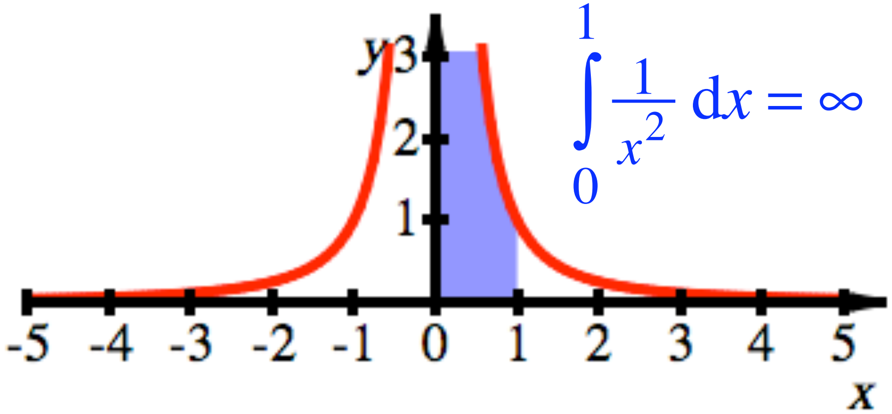


Heiße Grenze 0

$$f(x) = \frac{1}{x^2} \quad \Rightarrow \quad \int \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{x} + C \quad \Rightarrow \quad \int_s^1 \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{1} + \frac{1}{s}$$

$$\int_0^1 \frac{1}{x^2}\,\mathrm{d}x = \lim_{s\to 0}\left(\int_s^1 \frac{1}{x^2}\,\mathrm{d}x\right) = \lim_{s\to 0}\left(-1 + \frac{1}{s}\right)$$ divergiert!

Beispiel: $f(x) = \frac{1}{x^2}$



$\int_0^1 \frac{1}{x^2}\,\mathrm{d}x = \infty$

Heiße Grenze 0

$$f(x) = \frac{1}{x^2} \quad \Rightarrow \quad \int \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{x} + C \quad \Rightarrow \quad \int_s^1 \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{1} + \frac{1}{s}$$

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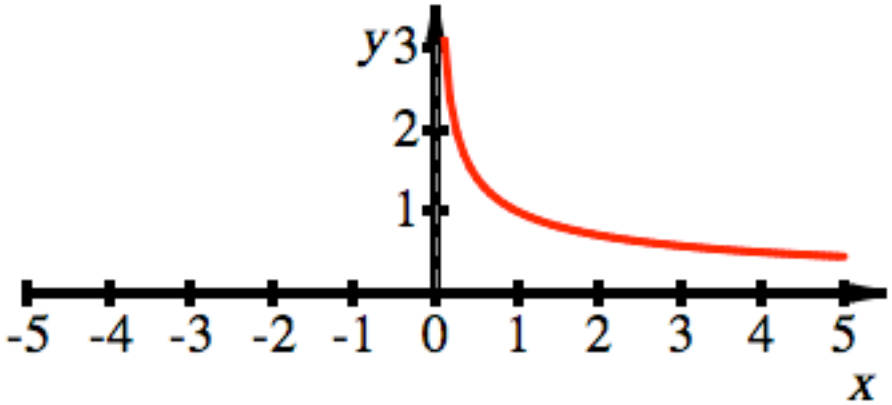
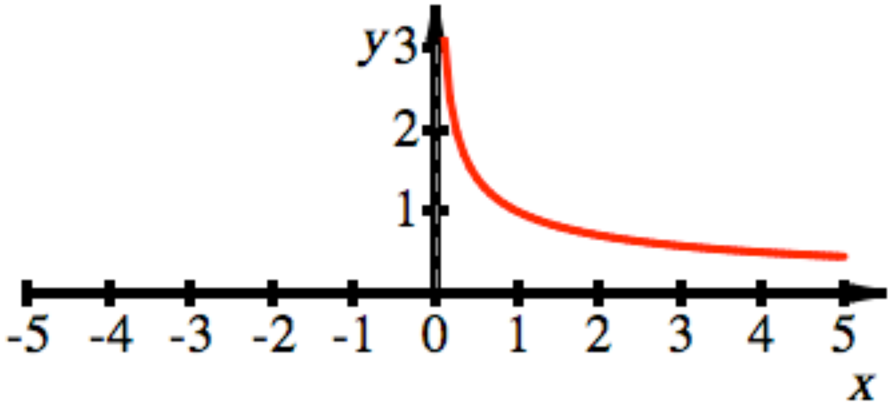
Beispiel: $f(x) = \frac{1}{\sqrt{x}}$



Beispiel: $f(x) = \frac{1}{\sqrt{x}}$

$$\int \frac{1}{\sqrt{x}}\,\mathrm{d}x = 2\sqrt{x} + C$$



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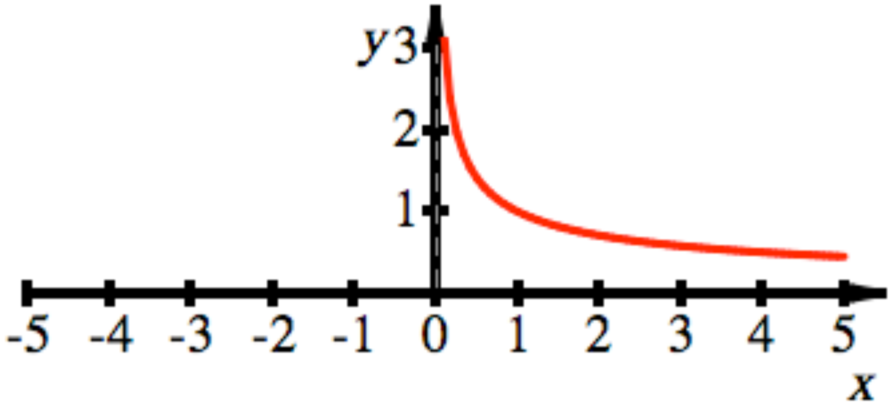
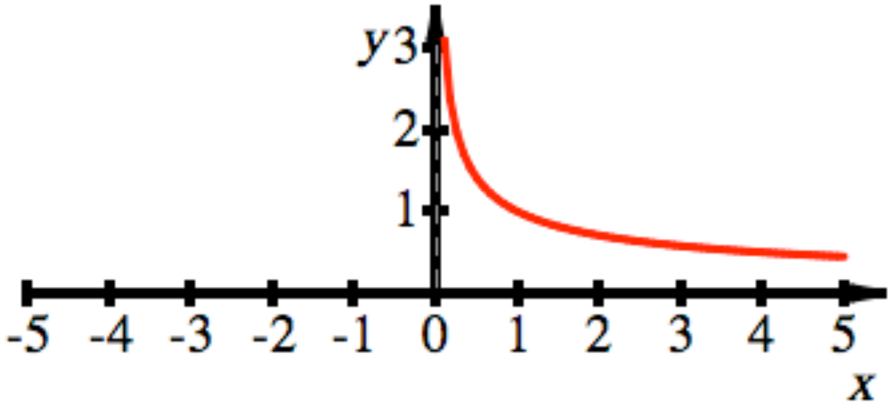


$$\int \frac{1}{\sqrt{x}}\,\mathrm{d}x = 2\sqrt{x} + C$$

Heiße Grenze +∞

$$\int_1^r \frac{1}{\sqrt{x}}\,\mathrm{d}x = 2\sqrt{r} - 2\sqrt{1}$$

Beispiel: $f(x) = \frac{1}{\sqrt{x}}$



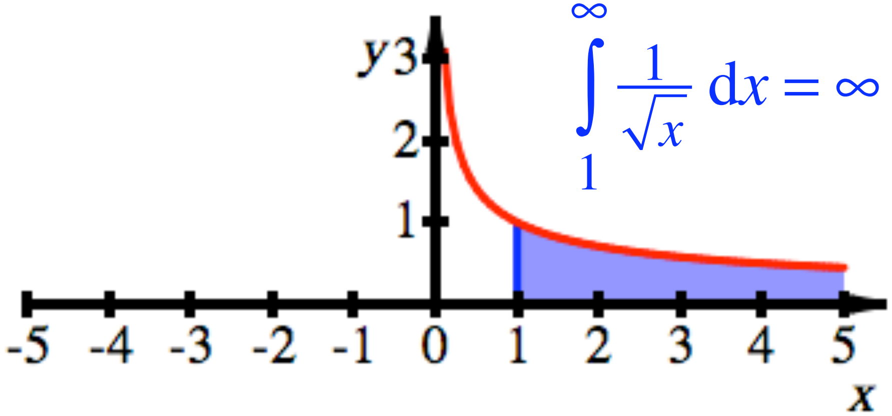
$$\int \frac{1}{\sqrt{x}}\,\mathrm{d}x = 2\sqrt{x} + C$$

Heiße Grenze $+\infty$

$$\int_1^r \frac{1}{\sqrt{x}}\,\mathrm{d}x = 2\sqrt{r} - 2\sqrt{1}$$

$$\Rightarrow \quad \int_1^\infty \frac{1}{\sqrt{x}}\,\mathrm{d}x = \lim_{r\to\infty}\left(\int_1^r \frac{1}{\sqrt{x}}\,\mathrm{d}x\right) = \lim_{r\to\infty}\left(2\sqrt{r} - 2\right)$$ divergiert!

Beispiel: $f(x) = \frac{1}{\sqrt{x}}$



$$\int \frac{1}{\sqrt{x}}\,\mathrm{d}x = 2\sqrt{x} + C$$

Heiße Grenze $+\infty$

$$\int_1^r \frac{1}{\sqrt{x}}\,\mathrm{d}x = 2\sqrt{r} - 2\sqrt{1}$$

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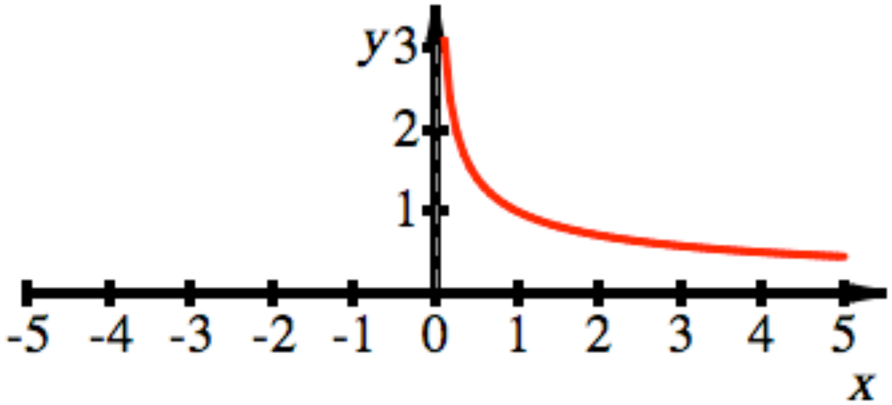
Beispiel: $f(x) = \frac{1}{\sqrt{x}}$



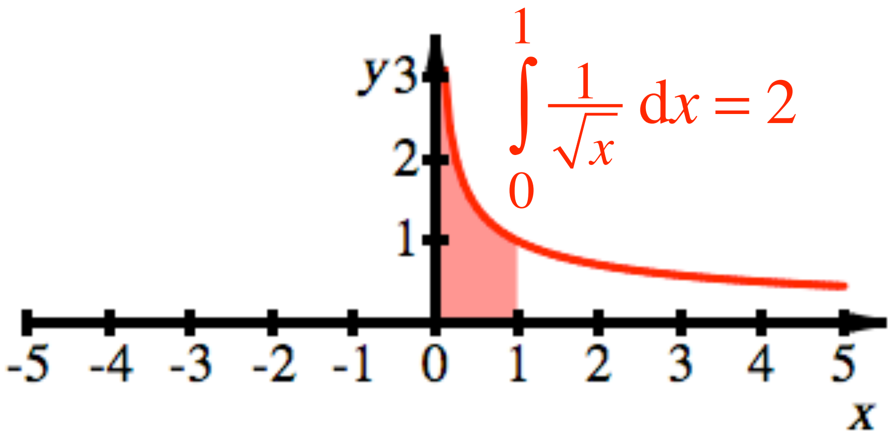
$$\int \frac{1}{\sqrt{x}}\, \mathrm{d}x = 2\sqrt{x} + C$$

Heiße Grenze 0

$$\int_s^1 \frac{1}{\sqrt{x}}\, \mathrm{d}x = 2\sqrt{1} - 2\sqrt{s}$$

$$\Rightarrow \quad \int_0^1 \frac{1}{\sqrt{x}}\, \mathrm{d}x = \lim_{s \to 0} \left(\int_s^1 \frac{1}{\sqrt{x}}\, \mathrm{d}x \right) = \lim_{s \to 0} \left(2 - 2\sqrt{s} \right) = 2$$

Beispiel: $f(x) = \frac{1}{\sqrt{x}}$



$$\int \frac{1}{\sqrt{x}}\,\mathrm{d}x = 2\sqrt{x} + C$$

Heiße Grenze 0

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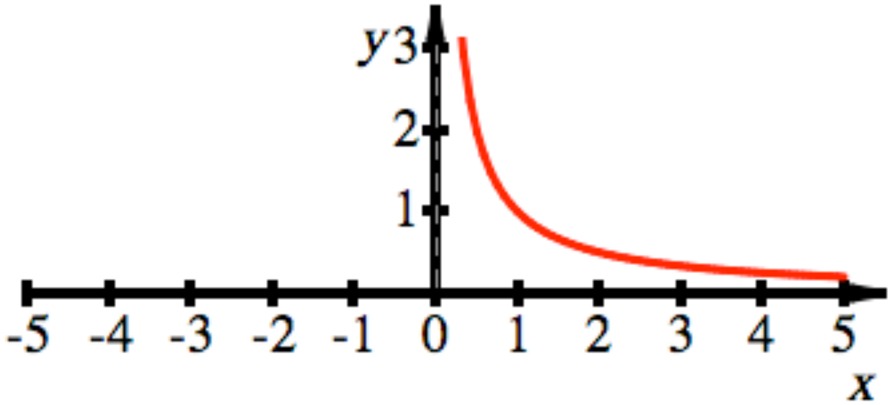
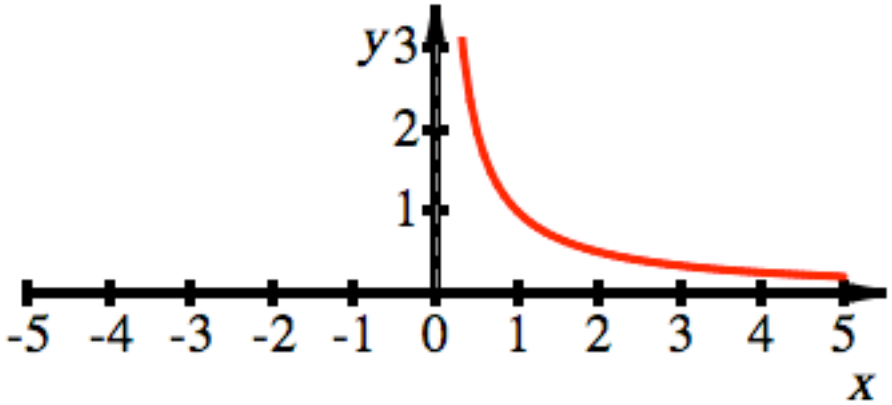
Beispiel: $f(x) = \frac{1}{x}$



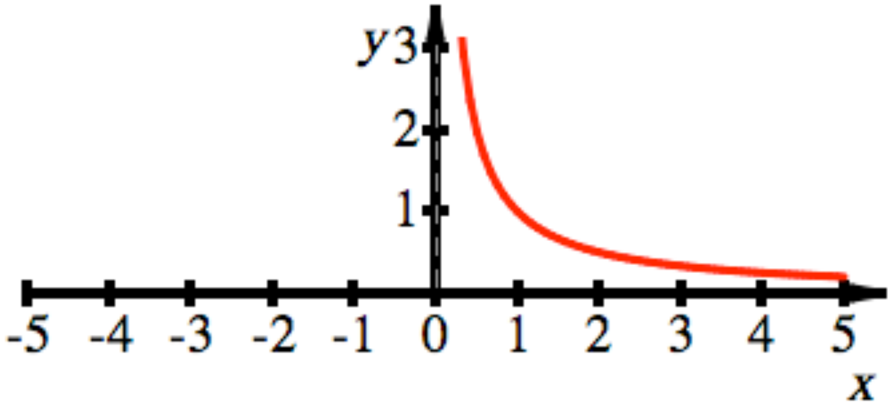
Beispiel: $f(x) = \frac{1}{x}$

$$\int \frac{1}{x}\,\mathrm{d}x = \ln(|x|) + C$$



Beispiel: $f(x) = \frac{1}{x}$

$$\int \frac{1}{x}\,\mathrm{d}x = \ln\left(|x|\right) + C$$



Heiße Grenze $+\infty$

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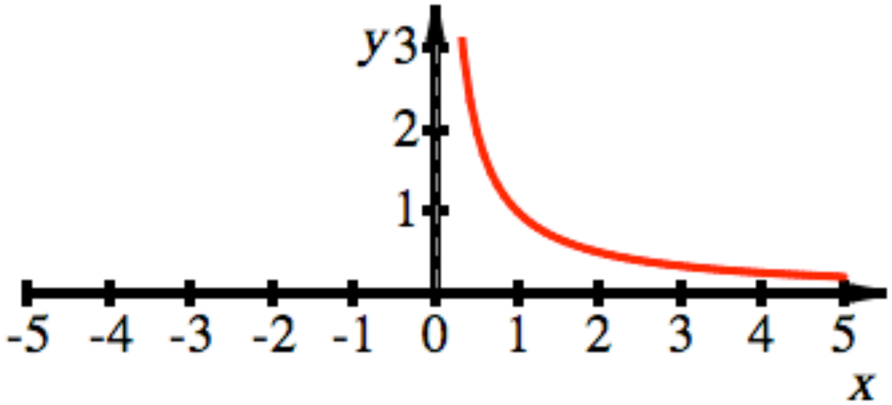
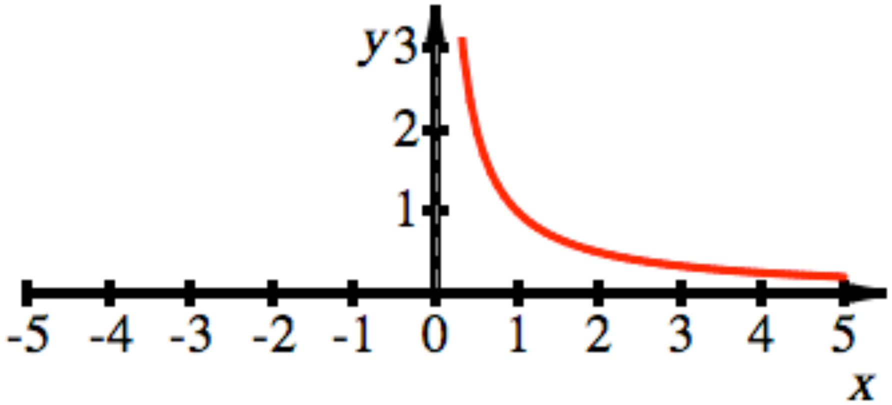


$$\int \frac{1}{x}\,\mathrm{d}x = \ln(|x|) + C$$

Heiße Grenze +∞

$$\int_1^r \frac{1}{x}\,\mathrm{d}x = \ln(|r|) - \underbrace{\ln(1)}_{=0} = \ln(|r|)$$

Beispiel: $f(x) = \frac{1}{x}$



$$\int \frac{1}{x}\,\mathrm{d}x = \ln(|x|) + C$$

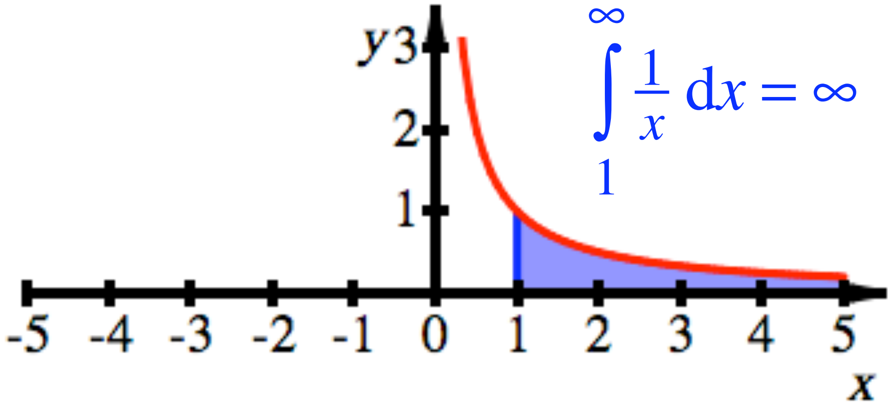
Heiße Grenze $+\infty$

$$\int_1^r \frac{1}{x}\,\mathrm{d}x = \ln(|r|) - \underbrace{\ln(1)}_{=0} = \ln(|r|)$$

$$\Rightarrow \quad \int_1^\infty \frac{1}{x}\,\mathrm{d}x = \lim_{r\to\infty}\left(\int_1^r \frac{1}{x}\,\mathrm{d}x\right) = \lim_{r\to\infty}\left(\ln(|r|)\right) \quad \text{divergiert!}$$

Beispiel: $f(x) = \frac{1}{x}$

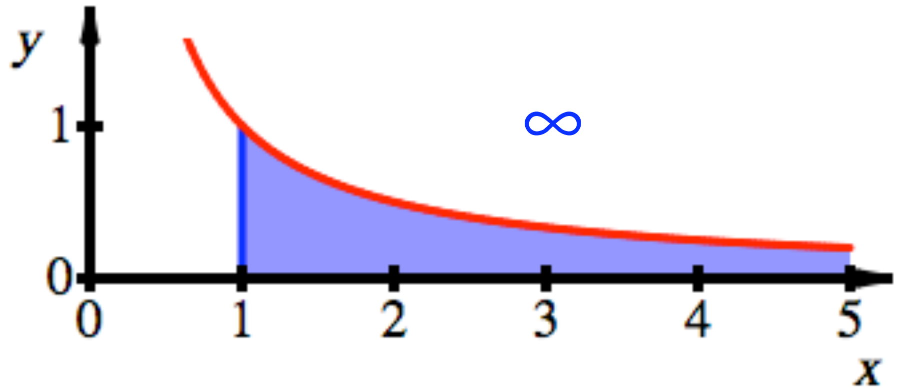
$$\int \frac{1}{x}\,\mathrm{d}x = \ln(|x|) + C$$



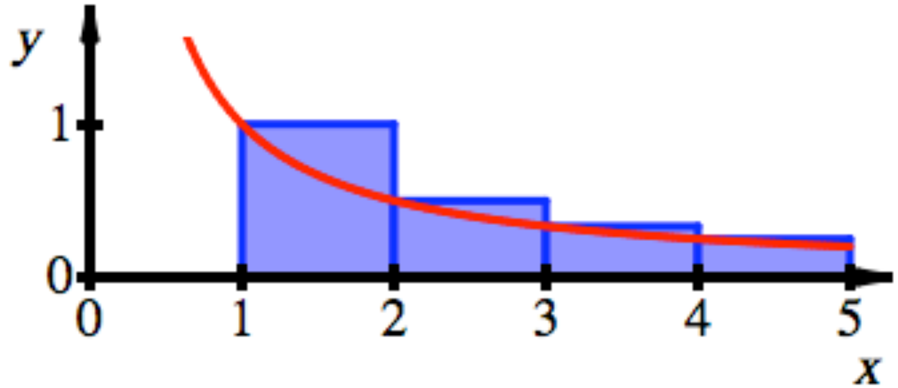
Heiße Grenze $+\infty$

$$\int_1^r \frac{1}{x}\,\mathrm{d}x = \ln(|r|) - \underbrace{\ln(1)}_{=0} = \ln(|r|)$$

$$\Rightarrow \quad \int_1^\infty \frac{1}{x}\,\mathrm{d}x = \lim_{r\to\infty}\left(\int_1^r \frac{1}{x}\,\mathrm{d}x\right) = \lim_{r\to\infty}\left(\ln(|r|)\right)$$ divergiert!



$$\int_1^\infty \frac{1}{x}\,\mathrm{d}x = \infty$$



$$1 + \frac{1}{2} + \frac{1}{3} + \cdots = \sum_{k=1}^{\infty} \frac{1}{k} > \infty$$

Die harmonische
Reihe divergiert